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# Argonne's global critical materials agent-based model (GCMat)

**Energy Systems, Decision and Infrastructure Science, and Strategic Security Sciences Divisions**

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# Argonne's global critical materials agent-based model (GCMat)

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**ERRATA**

| *Location* | *Error* | *Correction* |
|---|---|---|
| Page 4, paragraph 2, line 2 | **Figure** | Figure 2.1 |
| Page 4, paragraph 3, line 6 | **Figure** | Figure 2.1 |
| Page 4, paragraph 4, line 3 | **Figure** | Figure 2.1 |
| Page 8, paragraph 4, line 1 | **Figure** | Figure 2.1 |
| Page 8, paragraph 4, line 5 | as shown in **Error! Reference source not found**.. | as described in the following sections. |

# CONTENTS

# 1 INTRODUCTION

Several studies have identified rare earths as critical materials (DOE 2011, Nassar et al. 2015, NSTC 2016, Fortier et al. 2018). Although reasonably abundant in the Earth's crust, rare earths typically occur in low concentrations of mined ores (Van Gosen et al. 2017). Processes for recovering rare earth concentrates from these ores are complex and capital intensive (García et al. 2017, US EPA 2012). Further, lead times for deposit development, licensing, and construction are long, with reports of 10-15 years (Adamas 2016). China is a major player in the rare earths supply chain, both in production capacity and technology innovation (USGS 2019, Roskill 2018). In 2017, China supplied more than 80% of global rare earth oxide demand.

Rare earth elements (REEs) have unique magnetic, catalytic, and phosphorescent properties that significantly improve performance of a wide range of technologies. These technologies span aerospace, energy, telecommunications, electronics, transportation, defense, and other diverse applications. Consequently, disruptions in rare earth supply can have a significant societal impact. Estimating that impact requires understanding of the dynamics across the supply chain, from rare earth oxide extraction to end use application.

This report documents Argonne's Global Critical Materials model (GCMat), first described by Riddle et al. 2015. GCMat provides capabilities to explore supply chain dynamics and uncertainty under scenarios of demand growth or shrinkage, technology adoption, supply disruptions, and trade policies and mitigation strategies of new supply sources, product substitution, consumer thrifting, and stockpiling. Supply chain participants from rare earth mining through final demand are modeled as interacting agents who make market decisions independently as time progresses. Since the version documented in Riddle et al. 2015, GCMat has been expanded to cover additional REEs, derived products and supply chains that use these REEs, and includes new agent behaviors and modeling capabilities.

Section 2 provides a summary of the GCMat model design, including the structure of the model and key assumptions, section 3 summarizes methods used for model calibration and sensitivity analysis, and section 4 provides examples of model results.

# 2 GLOBAL CRITICAL MATERIALS (GCMat) MODEL

## 2.1 Background

The agent-based model documented in this report is an extended version of the GCMat model described in Riddle et al. 2015. Agent-based modeling offers a way to simulate dynamic economic markets that are composed of agents who (1) have complex decision-making behaviors, and (2) interact with and influence each other, possibly indirectly through market signals. The Repast Simphony agent-based modeling toolkit provides a framework for implementing and using agent-based models (North et al., 2013) and is the basis for GCMat.

The first step in GCMat's supply chain is rare earth mines. While rare earth mining and oxide separation facilities are not always co-located, mines in GCMat perform both steps and produce separated REOs.

Over 60 rare earth deposits are modeled using detailed data[1] on reserves, mineral compositions, capital and operating costs, and operational status. Each rare earth deposit is associated with a rare earth oxide mine and separation agent, who uses forecasted profitability and off-take contracting to inform decisions on when to start deposit development, when to start construction of a new mining facility, when to shut down a mining facility, or when to restart it. For mining facilities that are operating, mine managers make decisions on the quantity of rare earths to produce, the prices and quantities of rare earths offered to each potential buyer, and the management of their inventories. Chinese illegal mining activities are also included, and are modeled with a separate set of decision rules. Mine agents provide REOs to RE metal refiners and REO-containing product producers which supply products such as ceramics, catalytic converters, FCC catalysts and additives, optical glass & polish, phosphors, lasers and optical fiber.

Intermediate product producers in the supply chain are represented by additional agents in GCMat. Figure 2.1 illustrates the categories of agents in the supply chain, and the product flow between each supply and demand. Agent types represent regional aggregations for China, the US, and rest of world (ROW). These aggregations are used due to limited data availability on individual companies. Agents at each level act independently; vertical integration is not currently captured in the model.

RE metal refiners supply RE metals to RE metal-containing magnet producers, and RE metal-containing products such as metal alloys. As detailed in Riddle et al. (2015), neodymium-iron-boron (NdFeB) magnet producers supply magnets to wind turbine, electric and hybrid electric vehicle, and electric bike manufacturers, and to final demand agents. In the expanded model, these magnet producers also sell to producers of other NdFeB magnet containing products. REO-containing product producers and RE metal-containing product producers also supply end use producers (Figure 2.1).

Each of the producer agents have business decision-making behaviors for establishing contracts, updating price offers, buying and selling products, choosing technologies, setting production rates, managing inventories, and investing in new production capacities (Figure 2.1). Final demand agents respond to prices in their purchases of end-use products. Also represented in GCMat are government agents who enact regulations that may affect production levels and international trade and illegal supply agents who are not bound by government regulations and sell rare earth products at potentially disruptive prices and quantities.

The GCMat model runs are executed in time steps of one week, beginning in January 2010 and ending in January 2025. The model is calibrated to the first several years using historical price data. In each time step, price information is passed through the supply chain from mine managers to final demand sources, after which demand requests are passed back through to mine managers. Each agent then meets these demand requests if they can with available products, and products are passed back through to the supply chain. At each stage of the supply chain, the decision behaviors of agents determine product prices that nudge the markets towards a balance of supply and demand. Transactions that occur between the agents during the simulation are tracked, and regional results are reported for supply,

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[1] See Table 2.2 for sources of data

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demand, unmet demand, product substitution, and prices of all materials and products bought and sold throughout the supply chain.

By design, the GCMat-SM model captures market dynamics arising from the autonomous decision making and interactions of supply chain agents.  These market dynamics are explored in depth by modeling scenarios of interest, which may involve supply disruptions, government actions, and different expectations of demand, illegal supply, and technology change in the future.

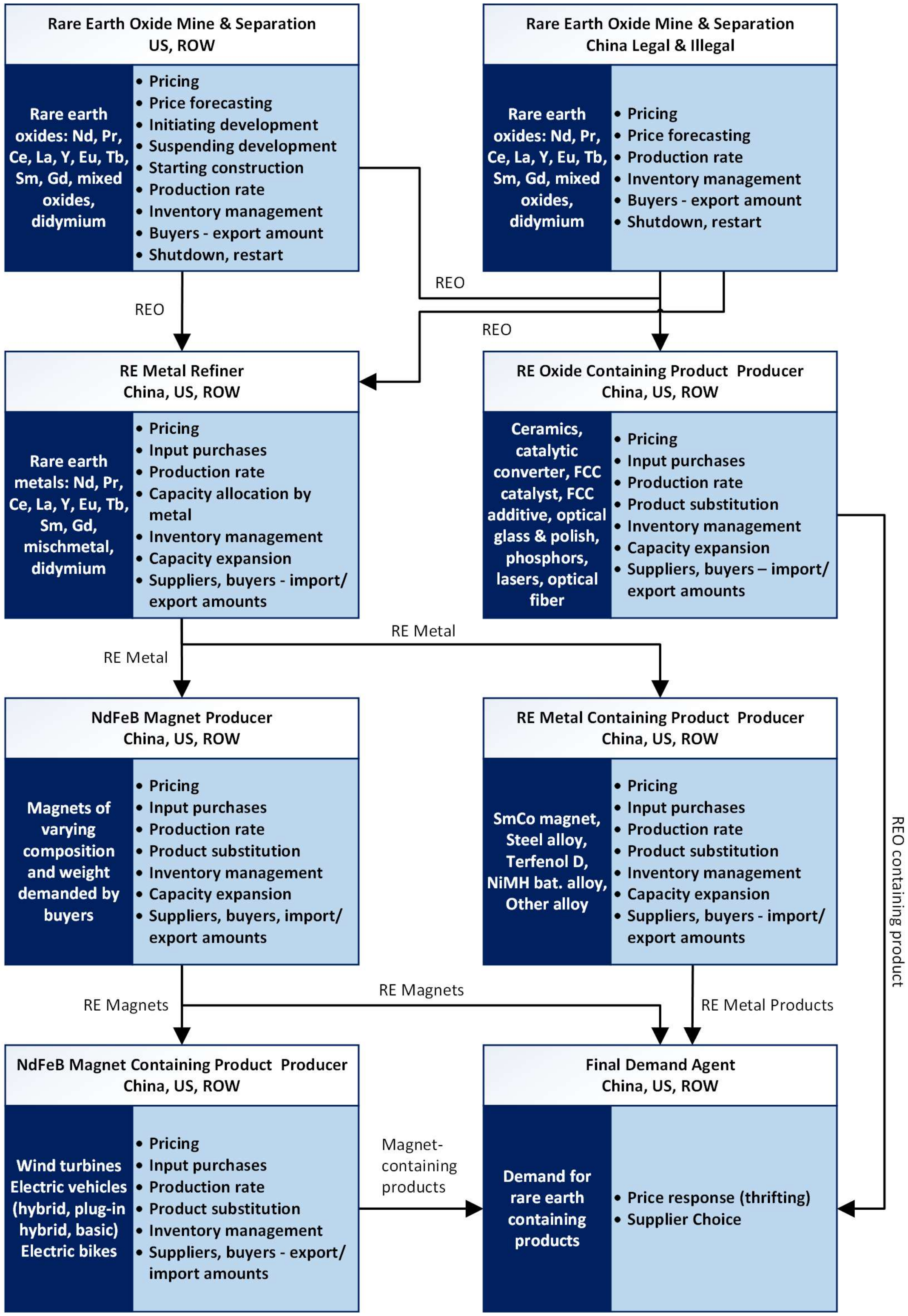



**Figure 2.1.** GCMat model agent behaviors and flows. Each large box represents an agent type, the dark blue inner boxes contain the types of products sold, and the light blue inner box provides a bullet list of behaviors for that agent.

## 2.2 Agents in GCMat

### 2.2.1 RE oxide mine and separation agents

These agents mine and separate rare earths, selling them in the form of oxides to RE metal refiners and RE oxide containing product producers. Rare earth oxides produced include Ce, La, Nd, Pr, Eu, Sm, Gd, Dy, Tb, Y oxides, as well as Nd/Pr (didymium) oxide, a mixed Ce/La oxide, and a mixed Ce/La/Nd/Pr oxide.

#### *2.2.1.1 Legal, non-China*

GCMat includes 72 rare earth oxide mining and separation agents outside of China, each of which is associated with a rare earth deposit that has a published mineral resource estimate. Four of these 72 mines are operating at the start of the model in 2010, others may open during the course of the model run, while others will never operate with current input data settings.

#### *2.2.1.2 Legal, China*

GCMat includes ten legal rare earth oxide mining and separation agents in China representing the 9 Chinese rare earth-producing provinces, Fujian, Guangdong, Guangxi, Hunan, Inner Mongolia, Jiangxi, Shandong, Sichuan and Yunnan, with two different mines in Jiangxi representing the Xunwu and Longnan deposits.

#### *2.2.1.3 Illegal, China*

GCMat includes ten Chinese illegal suppliers of rare earth oxides, one corresponding to each legal Chinese producer. Buyers include ROW metal refiners (primarily), as well as buyers of oxides for other end products that do not require refining (e.g. ceramics, phosphors, and catalysts). US buyers do not buy illegal oxides (though they may buy downstream products produced using illegal oxides).

### 2.2.2 RE metal refiners

GCMat includes three metal refiners, one from China, one from the U. S., and one from the rest of world, who purchase rare earth oxides and refine them into rare earth metals, and sell them to NdFeB magnet producers and RE metal containing product producers.

### 2.2.3 NdFeB magnet producers

GCMat includes three NdFeB magnet producers, one from China, one from the U. S., and one from the rest of world, who purchase rare earth metals and produce magnets from them, and sell them to NdFeB magnet containing product producers and final demand agents.

### 2.2.4 End use producers

End use producer agents purchase RE oxides, RE metals, or NdFeB magnets and generate products that can be purchased by final demand agents.

#### *2.2.4.1 RE oxide containing product producers*

GCMat includes 48 rare earth oxide containing product producers, including one producer of each product from China, U.S., and rest of world. These products include ceramic capacitors, Y ceramics, other ceramics, catalytic converters, FCC catalysts, FCC additives, other catalysts, polished glass, optical glass, other glass, LEDs, CFLs, LFLs, LCDs, lasers, and optical fiber.

#### *2.2.4.2 RE metal containing product producers*

GCMat includes 15 rare earth metal containing product producers, including one producer of each product from China, U.S., and rest of world. These products include SmCo magnets, Terfenol D, battery alloys, steel alloys, and other alloys.

#### *2.2.4.3 NdFeB magnet containing product producer*

GCMat includes 14 wind turbine producers, 14 electric and hybrid vehicle producers, and three electric bicycle producers. The wind turbine and vehicle producers roughly represent the top real-world wind turbine and vehicle producing companies, with further breakdowns by region if the producer operates in more than one region. Electric bicycle companies are aggregated by region. Wind turbine producers sell onshore and offshore wind turbines, and Electric vehicle producers sell BEVs (battery electric vehicles), HEVs (hybrid electric vehicles), and PHEVs (plug-in hybrid electric vehicles).

### 2.2.5 Final demand agents

GCMat includes 405 final demand agents, representing the final demand for all the different products modeled, with three regions, as well as military and civilian break-downs of the demand for each product.

## 2.3 Model schedule

Most of the agent types described in Figure 2.1 share common behaviors that can abstractly be categorized as suppliers, buyers, or final demand. The GCMat model architecture minimizes behavioral duplication in the code by grouping shared behaviors by the abstract behavioral types. As such, the overall model behavior is succinctly described by a sequence of ordered events that are applied by the abstract agent behavior category as described in the following sections. For example, at a single point in the model step sequence, all buyer agents update their demands for products from supplier agents. The general model step for this behavior does not determine how each of the buyer agents should update their demands, but only that this behavior is to be executed. The demand agents (e.g., end use producers, final demands, magnet producers, etc.) then execute the corresponding behavior in that step that is specifically tailored to the needed behavior by the agent.

### 2.3.1 Supplier agent update production, capacity, substitution

The following agent behaviors are called here:

- Production, inventory updating
- Capacity expansion
- Product substitution

### 2.3.2 Supplier agent update offers

The following agent behaviors are called here:

- Government export quotas
- Pricing
- Transaction costs

- Illegal production rates
- Buyers – export amounts (quantity offers)

#### 2.3.3 Buyer agent update demands

The following agent behaviors are called here:

- Input inventory targets
- Input purchases
- Price response
- Suppliers – import amounts

#### 2.3.4 Supplier agent update operations

The following agent behaviors are called here:

- Sales, inventory updating

#### 2.3.5 Supplier agent update product inventory targets

The following agent behaviors are called here:

- Update product inventory targets

#### 2.3.6 Supplier agent update deposit development and production decisions

The following agent behaviors are called here:

- Forecasting
- Deposit development
- Mine production capacity
- Production rates (except illegal)

### 2.4 Agent behaviors

#### 2.4.1 Government Export Quotas

Export quotas define a limit on sales from supplier agents for a list of specific products in a finite time period. Quotas are defined by a specific start and end time along with a separate basis period start and end time. There are two types of export quotas – historical Chinese export quotas and restrictions on exports due to some type of supply disruption. Chinese historical export quotas are set up to match the ones that actually existed. Export quotas for total rare earth oxides (TREO) were set to be 30,258 tonnes in 2010 and 30,246 tonnes in 2011. The 2010 quotas were separated into separate quotas for the first and second halves of the year to reflect the announcement of smaller-than-expected second half quotas. From 2012-2015, separate quotas were established for light rare earth oxides (LREO, including Nd, Pr, Ce, La) and medium/heavy rare earth oxides (HREO, including Sm, Eu, Gd, Dy, Tb, Eu. Y). The quotas for TREO, LREO, and HREO from 2010 through 2014 are shown in Table 2.1.

**Table 2.1: TREO, LREO, and HREO quotas in GCMat, 2012-2014**

| Year | TREO | LREO | HREO |
|---|---|---|---|
| 2010 | 30,258 | | |
| 2011 | 30,246 | | |
| 2012 | | 27,122 | 3,874 |
| 2013 | | 27,382 | 3,617 |
| 2014 | | 27,006 | 3,605 |

In addition, one set of disruption quotas was also added during the historical period: from Sept 21 to Nov 19, 2010, Chinese exports of all oxides and metals to ROW were restricted to 30% of exports at the start of the disruption, reflecting the cut-off of shipments to Japan that occurred during this period. In addition, we have set up the ability to implement a future disruption that may restrict exports of some products between some regions, with the details established by a set of parameters: $fysd$, the first year of the supply disruption, $lsd$, the length of the supply disruption and $sdser$, a set of parameters that determine the portion of legal sales between each pair of regions that is allowed to remain during the supply disruption. These quotas are set separately for each model step (week) and for each supplier and product, and are set to be a fixed portion of exports at the start of the disruption. Costs of illegal exports may also be increased during a supply disruption.

Sales Limits from Export Quotas

An export quota may cover multiple suppliers over multiple periods. The quota is distributed between producers in proportion to their exports during a basis period. For the historical quotas, the basis period is three years; for the disruption quotas it is one time step. If the quota covers an extended period of time, as with the historical Chinese export quotas, the amount that is available to export in each period is limited by a calculation that is aimed at making the quota be used up by the end of the quota period.

The export sales limit $ESL_t$ at each time step *t* is

$$ESL_t = min\left[TO_{t-1} \cdot mtot \cdot (OT_t - TO_{t-1}),\ SR_{t,max}\right]$$

Where $TO_{t-1}$ is the total offers lag, *mtot* is the movement towards offer target in period parameter, $OT_t$ is the current offer target, and $SR_{t,max}$ is the maximum sales rate:

$$SR_{t,max} = \frac{S_{t,remain}}{\Delta t}$$

Such that $S_{t,remain}$ is the remaining sales allowed at time *t* according to the export quota.

The offer target is defined as

$$OT_t = \frac{S_{t,remai}}{TR_{t,q}} \cdot min[oost_{max}, (1/SOO_t)]$$

Where $TR_{t,q}$ is the time remaining in the export quota, $oost_{max}$ is the maximum offer over sales target parameter and $SOO_t$ is the expected sales over offer at time $t$. The expected sales over offer is

$$SOO_{t,expected} = \begin{cases} esoo_{initial} & if\ TO_{t-1} = 0 \\ \dfrac{TS_{t-1}}{TO_{t-1}} & if\ TO_{t-1} > 0 \end{cases}$$

Where $esoo_{initial}$ is the initial expected sales over offer, and $TS_{t-1}$ is the total sales lag. Both the total offers and total sales lags are summed for all products included in the export quota for all product buyers covered by the quota:

$$TO_{t-1} = \sum_p \sum_b QO_{p,b,t-1}$$

Where $QO_{p,b,t-1}$ is the quantity offer of product *p* for buyer *b* at *t*-1. Similarly, the total sales lag is

$$TS_{t-1} = \sum_p \sum_b S_{p,b,t-1}$$

Where $S_{p,b,t-1}$ is the sales lag for product *p* for buyer *b*.

Government Export Quota Parameters

| Parameter | Description |
|---|---|
| $fysd$ | first year of supply disruption |
| $lsdy$ | length of supply disruption in years |
| $sdser$[1] | supply disruption share of exports remaining |
| $mtot$ | Movement towards offer target in period |
| $oost_{max}$ | Maximum offer over sales target |
| $esoo_{initial}$ | Initial expected sales over offer |

1. Separate input values may be set for each mine, source region, and destination region.

### 2.4.2 Transaction costs

Transaction costs in the form of government taxes and tariffs on legal product offers occur between buyers and producers in different regions. The base offer product price $P_{p,t}$ is multiplied by the cost of insurance and freight (CIF) rate and export taxes, both of which are dependent on the source and destination countries. The total buyer price is therefore

$$P'_{p,t} = P_{p,t}\left(1 + TC_{rs,rb,p,t}\right)$$

Where the total transaction cost (%) including CIF and export taxes, $TC_{rs,rb,p,t}$ is a function of the seller region (*rs*) and buyer region (*rb*) for each product *p*

$$TC_{rs,rb,p,t} = \left(1 + Tax_{rs,rb,p,t}\right)\left(1 + CIF_{rs,rb,p,t}\right)\left(1 + Tarf_{rs,rb,p,t}\right) - 1$$

Where $Tax_{rs,rb,p,t}$ is the export tax rate (%), $CIF_{rs,rb,p,t}$ is the CIF rate (%), and $Tarf_{rs,rb,p,t}$ is the tariff (%).

### 2.4.3 Forecasting

#### *2.4.3.1 Demand forecasting*

Demand forecasts are based on past trends in demand. The demand forecast made at time $t$ for the year $tf$ ($DF_{t,tf}$) is made by first estimating a long-run trend, $DFT_{t,tf}$, which is governed by its starting value, $DFT_{t,t}$, and its growth rate, $DFG_t$, each of which is updated each period.

$$DFT_{t,tf} = DFT_{t,t} \cdot (1 + DFG_t)^{(tf-t)}$$

$$DFT_{t,t} = (1 - dmaip)DFT_{t-1,t} + (dmaip)D_t$$

$$\ln(1 + DFG_t) = (1 - dgmaip)\ln(1 + DFG_{t-1}) + (dgmaip)\ln\left(\frac{DFT_{t,t}}{DFT_{t-1,t-1}}\right)$$

Demand is forecast to converge from its current level toward this long-run trend.

$$DF_{t,tf} = DFT_{t,tf} + dfcip^{(tf-t)/TS} \cdot \left(D_t - DFT_{t,t}\right)$$

Where

$$dmaip = 1 - e^{log(0.5)/dmah \cdot TS}$$

$$dgmaip = 1 - e^{log(0.5)/dgmah \cdot TS}$$

$$dfcip = 1 - e^{log(0.5)/dfchl \cdot TS}$$

When used for forecasting worldwide oxide demand, the following parameters are used

The model input parameters for the demand moving average half-life (*dmahl*), the demand growth moving average half-life (*dgmahl*), and the demand forecast convergence to trend half-life (*dfcl*) are given in the table below.

#### *2.4.3.2 Supply forecasting*

Supply is forecast to increase based on projections of when new deposits will come online. Production from China is assumed to be equal to the Chinese production quota for the specified time, which is assumed to be known. Each deposit that is actively being developed is projected to come online with probability $PD_{d,t,tf}$, which is calculated based on the development stage, $DS_{d,t}$, which ranges from 0 when development is initiated to 1 when the deposit is ready to produce. Deposits that are further developed having a greater chance of coming online.

$$PD_{d,t,tf} = \begin{cases} 0 & if\ deposit\ wouldn't\ be\ online\ by\ time\ tf \\ 0 & if\ deposit\ development\ has\ not\ been\ initiated \\ plwi & if\ deposit\ is\ in\ early\ development\ and\ would\ be\ online\ by\ time\ tf \\ 1 & if\ deposit\ is\ producing\ or\ under\ construction\ and\ would\ be\ online\ by\ time\ tf \end{cases}$$

$$SF_{p,t,tf} = \sum_{d \epsilon \bar{D}} PC_{d,p} \cdot PD_{d,t,tf}$$

#### 2.4.3.3 *Price forecasting*

Price forecasts made at time $t$ are for a set of future times, $tf \epsilon T = \{t, t+\Delta t, t+2\Delta t, \ldots, t+n\Delta t\}$.

$$PF_{t,tf} = \exp\left(\ln\left(PFM_{t,tf}\right) + PFR_{t,tf} \cdot \ln\left(DF_{t,tf}/SF_{t,tf}\right)\right)$$

$PFM_{p,t,tf}$ is the middle of a range of forecasts made just based on current price and the moving average of past prices, defined by:

$$\ln\left(PFM_{p,t,tf}\right) = \ln(PMAF_t) + \exp\left(-pfcmar\frac{\Delta t}{r}\right) \cdot \left(\ln\left(PF_{t,tf-\Delta t}\right) - \ln(PMAF_t)\right)$$

$$pfcmar = \frac{ln(0.5)}{pfcmahl}$$

$PFR$ (the price forecast range) determines how much effect the supply/demand balance has in each period:

$$PFR_{t,tf} = pfrsdi \cdot \Delta t \cdot \exp\left(-pfrr(tf - t)\right)$$

$$pfrr = \frac{ln(0.5)}{pfrh}$$

$PMAF$ is the moving average of past prices defined by:

$$PMAF_t = \frac{\sum_{i=1}^{t}(1 - pfmaip)^i\, P_{t-i}}{\sum_{i=1}^{t}(1 - pfmaip)^i}$$

$$pfmaip = 1 - e^{log(0.5)/pfmah \cdot TS}$$

where *pdrsdi* is the price forecast response to supply-demand imbalance, *pfrhl* is the price forecast range half-life, *pfcmahl* is the price forecast convergence to moving average half-life, and *pfmahl* is the price forecast moving average half-life.

Forecasting Parameters

| Parameter | Description |
|---|---|
| *plwi* | Forecast production likelihood when initiated |
| *dmahl* | Demand moving average half-life |
| *dgmahl* | Demand growth moving average half-life |
| *dfcl* | Demand forecast convergence to trend half-life |

| | |
|---|---|
| *pdrsdi* | Price forecast response to supply-demand imbalance |
| *pfrhl* | Price forecast range half-life |
| *pfcmahl* | Price forecast convergence to moving average half-life |
| *pfmahl* | Price forecast moving average half-life |

### 2.4.4 Deposit development decisions

The deposit development process, illustrated in Figure 2.2, determines how and if deposit developers initiate, continue, or cancel developments of mine deposits. All deposit development decisions account for uncertainty in revenue streams. Decisions to enter construction (when most funding is committed) is based on risk-averse evaluation of profitability; early development decisions are more risk neutral. This may lead to many deposits getting through early development phase with fewer advancing to construction. Profitability forecasts account for expected sales of some rare earths (e.g., Lanthanum and Cerium) being less than expected production. Advance contracts increase expected sales and reduce revenue uncertainty.

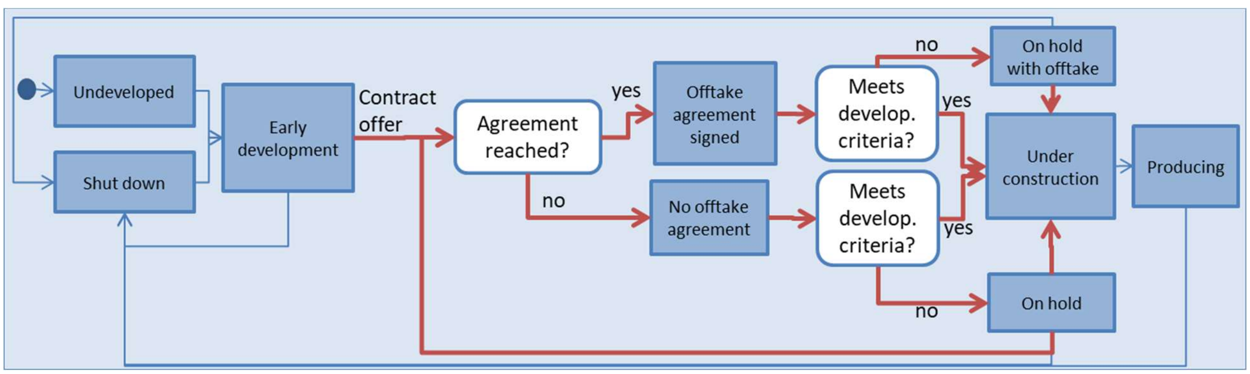



**Figure 2.2**. Flowchart showing decision process during deposit development

The full deposit development period (including early development and construction), if uninterrupted, is determined by the input parameter $ddp$ (deposit years to production). The construction period is determined by the input parameter $dcp$ (deposit construction stage in years).

#### *2.4.4.1 Offtake agreements*

Offtake agreements provide deposit developers with both 1) a guaranteed buyer, and 2) a risk-free revenue stream. Mining agents make choice of risk-free revenue versus uncertain revenue based on an explicit representation of price uncertainty in their decision-making process.

Offtake agreements are arranged before the construction phase (the final 2-3 years of development), when the majority of the financial resources are committed. Steps to arranging offtake agreements: 1) mines offer prices and max quantities, 2) metal refiners select desired quantities, and 3) mines choose from buyer requests.

First, the Mine agents propose offtake agreement in which the Price, quantity, and duration stay constant during the life of a contract. The contract length is specified by the parameter *dacl* (deposit advance contract length), prices and quantities are endogenously determined, and contracts can include multiple REOs, in any proportions. Initial price offers from mines are the higher of: 1) a markup from expected production cost, and 2) the risk-adjusted forecast price. Initial quantities offered are a fixed portion, determined by the input parameter *dmcpc* (deposit max contract Percent of capacity) of maximum production capacity.

Next, all buyers of oxides from US and ROW make demand forecasts using the demand forecasting rule from the 'Demand Forecasting' section with their own use of each oxide as the inputs to the demand forecast. They determine the maximum amount of each oxide they want to buy with long-term contracts, based on a user-specified portion of these demand forecasts, *mpdltc* (max percent desired in long term contracts) and what they already have tied up in contracts, as illustrated in Figure 2.3. Metal refiners select the lowest price option from either a set of available off-take offers or the short-term market price (based on forecasts), with a possible discount for long-term contracts based on the parameter *ltcp* (long term contract preference). Oxide buyers then offer a set of REO quantities they would buy at that price to the mine.

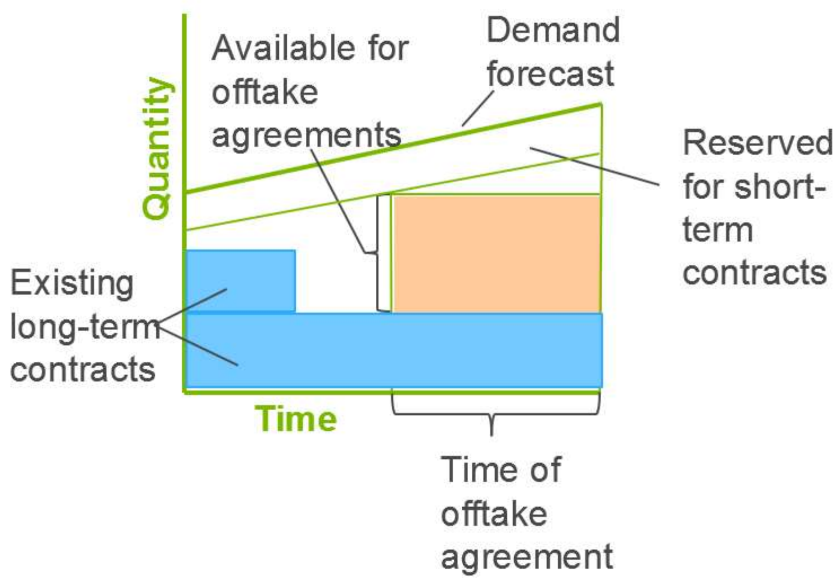



**Figure 2.3**. Buyer choice of desired quantity for offtake agreement

Finally, mines select a set of buyers. If a mine receives more product requests than they offered, they can choose to accept a portion of each offer, and choose the portions for each offer to maximize total oxides sold, as illustrated in Figure 2.4.

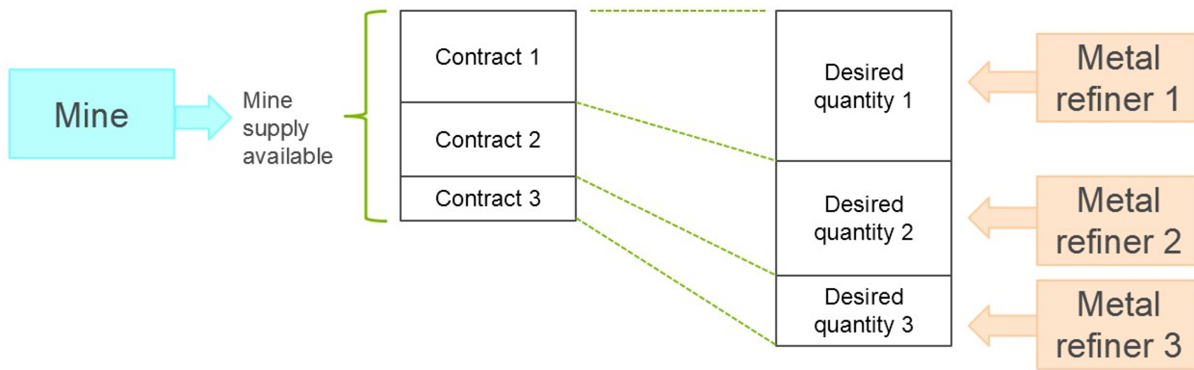



**Figure 2.4**. Supplier choice of offtake agreement contracts

### *2.4.4.2 Evaluating Profitability*

Deposits are evaluated regularly to determine if they would be profitable based on forecasts of prices during the period that the deposit would be operating. Mines use the price forecasting rule from the 'Price forecasting' section to forecast expected prices during mine operation. Oxide prices are forecast once per year for each year from the present until the end of the mine's expected operation period, MOP, which is the smaller of 20 years from opening and the amount of time it would take to fully deplete the deposit's resources. The supply forecast used in each deposit's price forecast assumes that deposit will be operating, so that larger deposits may expect to see a larger decrease in price after they come online. Prices of the mining region the deposit is in are used instead of worldwide prices.

<u>Price Forecast Uncertainty</u>

Uncertainty ranges are added to price forecasts based on historical price volatility, and are applied to the mean forecasts described above. The amount of uncertainty increases the farther into the future following an Ornstein Uhlenbeck process, which leads to the amount of uncertainty eventually approaching a steady state. The parameters of the Ornstein Uhlenbeck process are estimated separately for each oxide using historical price data.

<u>Net Present Value</u>

Given a set of price forecasts, the net present value of constructing and operating the mine is.

$$NPV = \sum_{tf \epsilon MOP} \exp(-dr \cdot tf)\left(\left(\sum_p S_{p,tf} PF_{p,t,tf}\right) - OC_{tf}\right) - \sum_{tf \epsilon CP} \exp(-dr \cdot tf)\left(\sum_p CC_{tf}\right)$$

Production is assumed to be at full capacity while the mine is operating, but sales may be less than production: $S_{p,tf} = desps_p \cdot PC_{p,tf} = desps_p \cdot PC_{TREO} \cdot OS_p$, for each $tf$ while the deposit is producing, and zero otherwise. The parameters $desps_p$ (deposit expected shares of production sold) are different for different oxides depending on their presumed level of demand. Operating costs, $OC_{tf} = OCR \cdot PC_{TREO}$, during each year the deposit is producing and 0 otherwise. Capital costs are evenly distributed over the construction period, so $CC_{tf} = \frac{TCC}{dcp}$ during construction and 0 otherwise.

$TCC$ (total capital cost), $OCR$ (operating cost ratio), $PC_{TREO}$ (production capacity when producing) and $OS_p$ (oxide share) are defined in the input data. $dcp$ is the deposit construction period, and $PR_{TREO}$ is the production rate for all oxides.

Risk Aversion

Deposit development decisions account for the decreased risk from a certain revenue stream from offtake agreements compared to an uncertain revenue stream from short term markets. This is done using utility theory; rather than comparing NPV of profits, we compare expected utility of different choices. This allows us to capture different risk profiles and attitudes. For example, risk-averse decision makers prefer a guaranteed revenue stream over a risky one with the same expected return.

$NPV$ is a random variable that accounts for uncertainty in prices not covered by contracts. We can evaluate if the expected value $E[u(\boldsymbol{NPV})]$>0, where $u(\cdot)$ is a risk-averse utility function. We use a constant relative risk aversion (CRRA) utility function, $u(c) = \begin{cases} \frac{NPV^{1-\alpha}-1}{1-\alpha} \; if \; \alpha\epsilon[0,1) \cup (1,\infty) \\ \ln(NPV) \; if \; \alpha = 1 \end{cases}$ so that scaling all prices by the same factor will not affect the results. Expected utility is calculated by averaging the results of Monte Carlo sampling from the distribution of $NPV$. The risk aversion parameter, $\alpha$, depends on the stage at which deposit development is being evaluated.

#### *2.4.4.3 Initiating development*

Deposit development can be initiated if the time is within the development period ($ddp$) of the deposit's earliest possible start date, which is specified in the input data. After that time, they regularly evaluate whether it would be profitable based on forecasts of prices during the period that the deposit would be operating. If the expected utility of developing and operating the mine is found to be > 0 using a risk-aversion parameter of $\alpha = dafi$ (deposit alpha for initiation) then they initiate early development.

#### *2.4.4.4 Suspending development*

Deposits continue to be evaluated regularly within the early development period, using updated price forecasts and a risk-aversion parameter of $\alpha = dafed$ (deposit alpha for early development). If at any point, their expected utility falls below 0, they suspend development. If they decide to suspend development, they can resume at any point if the conditions for initiating development are met, but their development is set back by $dsfa$ (deposit setback from abandoning) years.

#### *2.4.4.5 Starting construction*

To proceed to the construction phase, a deposit is reevaluated to determine whether it is profitable using a higher risk aversion parameter, $\alpha = dafec$ (deposit alpha for entering construction). Before making this evaluation, offtake agreements may be set up as described above to help reduce risks. Once in the construction phase, they proceed to bringing the deposit online without reevaluating profitability.

#### *2.4.4.6 Shutdown, restart*

Once a deposit has begun producing, they may be shut down if two criteria are met. First, they must accumulate enough losses to exceed $dclbs$ (deposit cumulative losses before shutdown) times their annual operating costs. Second they reevaluate future profitability using $\alpha = dafs$ (deposit alpha for shutdown) and shut down if their expected utility is less than zero.

Deposit Developer Parameters

| Parameter | Description |
| --- | --- |
| $ddp^1$ | Deposit years to production |
| $dcp^1$ | Deposit construction stage in years |
| $dacl$ | Deposit advance contract length |
| $dmcpc$ | Deposit max contract percent of capacity |
| $mpdltc^1$ | Max percent desired in long term contracts |
| $ltcp$ | Long term contract preference |
| ${desps_p}^2$ | Deposit expected shares of production sold |
| *dafi* | Deposit alpha for initiation |
| $dafed$ | Deposit alpha for early development |
| $dsfa^1$ | Deposit setback from abandoning |
| $dafec$ | Deposit alpha for entering construction |
| $dclbs^1$ | Deposit cumulative losses before shutdown |
| $dafs$ | Deposit alpha for shutdown |

1. Separate input values for each region.
2. Separate input values for each oxide.

### 2.4.5 Capacity expansion

#### *2.4.5.1 Mine production capacity*

For mine agents, capacity expansion is driven by deposit development decisions.

<u>Capacity expansion</u>

Capacity expansion is determined by the deposit developer agents. A mine manager's total production capacity is defined by:

$$PC_{p,t} = \sum_{d} PC_{d,p,t}$$

Where $PC_{d,p,t}$ is the production capacity for deposit *d*.

#### *2.4.5.2 NdFeB magnet-containing product producer production capacity*

NdFeB magnet-containing product producers have no capacity limits, they are always able to produce enough to meet demand if there are enough magnets available.

#### *2.4.5.3 Illegal oxide supplier production capacity*

Illegal supplier production capacity, $PC_{TREO,t}$, is set to be a fixed multiple of the government target for illegal supply, $GT_{TREO,t}$, which is specified in the input data.

$$PC_t = icmit \cdot GT_{TREO,t}$$

$icmit$ is the illegal capacity multiple of illegal target.

#### *2.4.5.4 Capacity expansion (all other agents)*

When producer agents update their offers, they also update their production capacities by summing the production capacity additions that are online. Capacity additions are only scheduled when the demand forecast for new capacity exceeds the sum of the current production capacity and capacity under development times the capacity utilization to initiate new capacity development (*cuincd*) input parameter.

The total production capacity for product *p* is given by

$$PC_{p,t+\Delta t} = PC_{p,t} + \sum CA_{p,t+\Delta t}$$

where $CA_{p,t+\Delta t}$ is the capacity addition size for product *p,* at time $t + \Delta t$. The time lag $\Delta t$ is the new capacity years to production input parameter (*ncytp*). For producer agents with multiple products, the total production capacity for each product is separate and indexed by the product name. Magnet producers and metal refiners calculate the total production capacity as above, however, this is the total production capacity for the producer agent, e.g. for magnet producers it is the production capacity of all magnets.

Capacity additions are either exogenous, which are preset model inputs that fix the production capacity of a specific producer agent at a specific time or endogenous, which are determined by the producer agents according to the demand forecast. Endogenous capacity additions are calculated as

$$CA_{p,t+\Delta t} = \frac{EDM_{p,t+\Delta t}}{cut_{new}} - (PC_{p,t} + CUD_{p,t})$$

Where $EDM_{p,t+\Delta t}$ is the expected maximum demand during the period from when the capacity would be added until $tbncs$ (time basis for new capacity size) years later, $cut_{new}$ is the new capacity utilization target input parameter, and $CUD_{p,t}$ is the capacity under development.

Capacity Expansion Parameters

| Parameter | Description |
|---|---|
| $icmit$ | Illegal capacity multiple of illegal target |
| $cuincd$ | Capacity utilization to initiate new capacity development |
| $ncytp$ | New capacity years to production |
| $tbncs$ | Time basis for new capacity size |
| $cut_{new}$ | New capacity utilization target |

### 2.4.6 Product substitution

#### *2.4.6.1 Product substitution for RE oxide and metal-containing product producers*

Several agents can use product substitution or thrifting to adjust the amount of rare earths used in their production process, or shift between the use of different rare earths. Product substitution rules are defined for LFL producers, CFL producers, FCC catalyst producers, FCC additive producers, glass polish producers, battery alloy producers, steel alloy producers, magnet producers, wind turbine producers, electric vehicle producers and electric bicycle producers. The rules for the following agents use the same updating rule:

- LFL producers can adjust the amount of tri-phosphor used per LFL bulb, while keeping the rare earths used in the same proportions
- CFL producers can adjust the amount of tri-phosphor used per CFL bulb, while keeping the rare earths used in the same proportions
- FCC catalyst producers can adjust the amount of La oxide used per tonne of FCC catalyst
- FCC additive producers can adjust the amount of Ce oxide used per tonne of FCC additive
- Glass polish producers can shift between using Ce oxide and a Ce/La mixed oxide
- Battery alloy producers can shift between using La metal and Ce/La/Nd/Pr mischmetal
- Steel alloy producers can shift between using Ce/La and Ce/La/Nd/Pr mischmetals

For each of these producers a minimum and a maximum value is defined in the input data that limits the amount of substitution that can occur. Each producer can choose to produce using a technology that can be expressed as a weighted average of this minimum and maximum options, based on the relative costs and benefits of producing using the minimum and maximum technologies. The substitution rule effectively allows the fractional *shares* of technology inputs to vary as long as the actual input flow remains between the min and max values.

Whenever a producer updates its list of available production technologies, it will check if a substitution rule exists for the technology and calculate the minimum technology cost $CT_{min,T,t,R}$ and maximum technology cost $CT_{max,T,t,R}$ for technology *T* at time *t*, in region *R*.

The input amount to each technology option $TA_{i,t}$ for input *i* at time *t* with a substitution rule is then updated according to:

$$TA_{i,t} = ST_{min,t} \cdot STA_{min,i,t} + ST_{max,t} \cdot STA_{max,i,t}$$

Where $ST_{min,t}$ and $ST_{max,t}$ are the substitution technology shares for the minimum and maximum technology substitution rule, and $STA_{min,i,t}$ and $STA_{max,i,t}$ are the input amounts for input *i* for the minimum and maximum technology substitution rule. Substitution technology shares are calculated as

$$ST_{max,t} = ST_{max,t-1} + \left(EST_{max,t} - ST_{max,t-1}\right) \cdot MTE$$

$$ST_{min,t} = 1 - ST_{max,t}$$

$$EST_{max,t} = \frac{e^{psercd*\ln(IC_{max,t})}}{e^{psercd*\ln(IC_{min,t})} + e^{psercd*\ln(IC_{max,t})}}$$

Where $EST_{min,t}$, $EST_{max,t}$ are the min and max equilibrium shares, $IC_{min,t}$ and $IC_{max,t}$ are the costs of producing the product using the minimum and maximum technologies, $psercd$ is the production share equilibrium response to cost differential, and $MTE$ is the movement towards equilibrium production shares in the period, defined as:

$$MTE = 1 - e^{log(0.5)/psechl \cdot TS}$$

Where $psechl$ is the production share equilibrium convergence half-life, and *TS* is the number of time steps per year.

Product Substitution for Oxide and Metal-containing Product Producers Parameters

| Parameter | Description |
|---|---|
| *psercd* | Production share equilibrium response to cost differential |
| $psechl^1$ | Production share equilibrium convergence half-life |
| *TS* | Time steps per year |

1. Separate input values for magnet producers and other producers.

### *2.4.6.2 Product substitution for magnet producers*

The substitution between Dy and Nd/Pr is very similar to the general substitution rule for oxide and metal containing product producers, with a target Dy percent set for each use of magnet and time (based on input data assimilated from Steve Constantinides, Adamas, and Roskill). Maximum and minimum values, $STA_{min,i,t}$ $STA_{max,i,t}$ are set to be $midpm$ (minimum Dy percent multiplier) and $madpm$ (maximum Dy percent multiplier) times the target levels. Higher Dy contents within this range are assumed to have a benefit of $bod$ (benefit of Dy) in $/kg of Dy. The result of the substitution rule calculation is an amount of Dy per magnet, $DyPM_{p,t}$ for each type of magnet, *p*, at each time *t*.

The remaining rare earth content of the magnets,

$$NdPrPM_{p,t} = 0.31 - DyPM_{p,t}$$

is then assigned to a combination of didymium (Nd/Pr), Nd and Pr metals.

It is assumed that didymium will be a better deal in the long-run than mixing Nd and Pr, so if the desired amount of Pr in a magnet is more than the amount of Pr in the purchased didymium, then producers use a mix of Pr and didymium; if the desired amount of Pr is less, then they use a mix of Nd and didymium. In practice, producers almost always use a combination of Nd and didymium

The maximum praseodymium from didymium, *MPFD*, id defined as

$$MPFD_{p,t} = PPD_t \cdot NdPrPM_{p,t}$$

Where $PPD_t$, or Pr per didymium, is the average composition of didymium purchased by the magnet producer.

The cost of increasing the amount of Pr per magnet (PrPM) is based on the relative price of Di compared to Nd if $PrPM \; < \; MFPD$, and the relative cost of Pr compared to Di if $PrPM > \; MFPD$.

In addition, costs of changing the amount of Pr per magnet are included. The marginal costs are assumed to increase linearly with the amount of the change, in addition to a fixed cost of making any change. The change costs are intended to capture the indirect cost to the magnet producer resulting from having to changes product specifications, in addition to any direct costs of making the change. The marginal costs, $\frac{dC}{dPrPM}$, are specified as follows:

$$\frac{dC}{dPrPM} = IC(PrPM) + VCC(PrPM) + FCC(PrPM)$$

$$IC(PrPM) = \begin{cases} \dfrac{(P_{Di,t} - P_{Nd,t})}{PPD_T} & if\ PrPM < MPFD_{p,t} \\ \dfrac{(P_{Pr,t} - P_{Di,t})}{1 - PPD_t} & if\ PrPM > MPFD_{p,t} \end{cases}$$

$$VCC(PrPM) = pbccs \cdot (PrPM_{t-1} - PrPM)$$

$$FCC(PrPM) = \begin{cases} 0 & if\ PrPM < PrPM_{t-1} \\ pbcc & if\ PrPM > PrPM_{t-1} \end{cases}$$

Where $IC(PrPm)$ are input costs, $VCC(PrPM)$ are variable change costs and $FCC(PrPM)$ are fixed change costs of increasing PrPM; $P_{Di}$, $P_{Nd}$, and $P_{Pr}$ are the prices of Didymium, Nd and Pr metals, $pbcc$ is the Pr benefit change cost, and $pbccs$ is the Pr benefit change cost slope.

Benefit curves are based on how close Pr per magnet is to target levels: marginal benefit:

$$\frac{dB}{dPrPM} = pbs \cdot (PRBXI_{p,t} - PrPM)$$

Where $PRBXI_{p,t}$ is Pr benefit x-intercept, which is defined separately for each type of magnet and time, based on input data, and $pbs$ is the Pr benefit slope.

The target Pr content per magnet, $TPrPM_{p,t}$, is calculated that maximizes revenues minus costs, given these cost and benefit curves.

The final Pr content per magnet is between its lagged value and the new target value:

$$PrPM_{p,t} = PrPM_{p,t-1} + MTE \cdot (TPrPM_{p,t} - PrPM_{p,t-1})$$

Where $MTE$ is movement to equilibrium and is calculated as

$$MTE = 1 - e^{log(0.5)/pctc \quad \cdot TS}$$

Where $pctchl$ is Pr content target convergence half-life.

Product Substitution for Magnet Producers Parameters

| Parameter | Description |
|---|---|
| *pbs* | Pr benefit slope |
| *pctchl* | Pr content target half life |
| *pbcc* | Pr benefit change cost |
| pbccs | Pr benefit change cost slope |
| *midpm* | Minimum Dy percent multiplier |
| *madpm* | Maximum Dy percent multiplier |
| *bod* | Benefit of Dy |
| *TS* | Time steps per year |

##### *2.4.6.3 Technology choice for NdFeB-containing product producers*

Product line development timeline

A new product line is initiated each year. When it is initiated, a technology is chosen from the set of available technologies. For wind turbine producers, the technology options are DFIG turbines that don't use NdFeB magnets, direct drive turbines that use large NdFeB magnets, hybrid turbines that use smaller NdFeB magnets, and Dy-free turbines that use large NdFeB magnets that contain no Dy. Electric vehicle producers choose between technologies that use two NdFeB magnets, one NdFeB magnet, NdFeB magnets with reduced Dy content, or induction motors that don't use NdFeB magnets. Electric bicycle producers can use a standard technology, one with a smaller motor, one with reduced Dy content, and one with no NdFeB magnet. After a development period of $t_P$ years, production of the new product line begins. Development continues for $t_P$ years.

Technology choice

When a new product line initiates development, a technology is randomly chosen with probabilities that depend on the relative production costs ($C_{pc,T,t}$) with each technology option

$$p(TC_{pc,pl,t} = \bar{T}) = \frac{\left(\frac{1}{C_{pc,\bar{T},t}}\right)^{w_{TC}}}{\sum_{to}\left(\frac{1}{C_{pc,T,t}}\right)^{w_{TC}}}$$

Where $w_{TC}$ is a weight that determines the probability of picking lower-cost technologies; a higher value means a higher probability of picking the lowest cost technology.

### 2.4.7 Inventory management

##### *2.4.7.1 Input inventory targets*

A risk-based input inventory target is calculated as the product of the inventory target in years $ITY_{p,t}$ and the input desired for production forecast, $IDPF_{p,t}$:

$$IT_{p,t} = ITY_{p,t} \cdot IDPF_{p,t}$$

The inventory desired for production forecast ($IDPF_{p,t}$) is based on the inputs desired for production ($IDP_{p,t}$), which in turn depends on production rates ($PR_{p,t-1}$), and on the technology relationships $n_{p,i}$, the amount of input $i$ needed to produce one unit of product $p$. The forecasting is done using the same approach as for demand forecasts, projecting into the future based on trends in past usage, as described in the 'Demand forecasting' section. The forecasting parameters used are the demand moving average half-life (*dmahl*) and the demand growth moving average half-life (*dgmahl*), and the input use forecast years into the future used (*iufyif*).

$$IDP_{p,t} = \sum_{p2} n_{p2,p} \cdot PR_{p2,t-1}$$

The inventory target in years $ITY_{p,t}$ is:

$$ITY_{p,t} = ITY_{base,p,t} + \left(1 - e^{-c \cdot Risk_{p,t}}\right) \cdot (iirmt - iitby)$$

Where *iitby* is the region and producer specific input inventory target base (without builds or risk adjustment) parameter, in years of expected input needs for production, *iitrmt* is the input inventory risk max target parameter, *c* is the input inventory target manager base inventory target calculation risk coefficient. The user-defined base input inventory target in years, $ITY_{base,p,t}$ is generally defined as:

$$ITY_{base,p,t} = iitby$$

and more specifically under the following time periods:

$$ITY_{base,p,t} = \begin{cases} iitby + tidby - tdddy & t > idet \\ iitby + tidby - tdddy + tdddy \cdot \left(\dfrac{idet - t}{idet - idst}\right) & t > idst \\ iitby + tidby & t > ibet \\ iitby + tidby \cdot \left(\dfrac{t - ibst}{ibet - ibst}\right) & t > ibst \end{cases}$$

The time-based input parameters are:

- *idet*: the input inventory drawdown end time which is the time at which the gradual drawdown in inventories following the 2010-2011 price spike ends.
- *idst*: the input inventory drawdown start time which is the time at which the gradual drawdown in inventories following the 2010-2011 price spike begins.
- *ibet*: the input inventory build end time which is the time at which the inventory build that helped cause the 2010-2011 price spike ends.
- *ibst*: the input inventory build start time which is the time at which the inventory build that helped cause the 2010-2011 price spike begins.

Additional parameters include the target increase during build in years, *tidby*, which is the product and region-specific amount that input inventory targets increase during the inventory build that helped cause the 2010-2011 price spike. The target decrease during drawdown in years, *tdddy*, is

$$tdddy = tidby + idob$$

Where *idob* is the inventory drawdown over build input parameter.

The risk is

$$Risk_{p,t} = Risk_{p,t-1} \cdot e^{-r \cdot TS} + UDR_{p,t-1} + w_1 \cdot UDR_{region,p,t-1} + w_2 \cdot UDR_{world,p,t-1}$$

where *TS* is the time steps per year input parameter, $r$ is the input inventory risk parameter, $w_1$ is the input inventory risk region group weight, and $w_2$ is the input inventory risk world weight.

The producer-specific unmet demand ratio (lag) for input *p* accounts for the difference between the producer's input demand in the last time $D_{p,t-1}$ and the actual amount of input sales received $S_{p,t-1}$ by the producer:

$$UDR_{p,t-1} = \frac{D_{p,t-1} - S_{p,t-1}}{D_{p,t-1}}$$

The unmet demand ratio for world ($UDR_{world,p,t-1}$) and regional ($UDR_{region,p,t-1}$) product demands and sales are calculated via the same equations, except that the respective quantities for demands and sales are used:

$$UDR_{world,p,t-1} = \frac{D_{world,p,t-1} - S_{world,p,t-1}}{D_{world,p,t-1}}$$

$$UDR_{region,p,t-1} = \frac{D_{region,p,t-1} - S_{region,p,t-1}}{D_{region,p,t-1}}$$

Input Inventory Target Parameters

| Parameter | Description |
|---|---|
| *Iitby*[1] | Region and producer specific input inventory target |
| *Dmahl* | Input inventory demand moving average half-life |
| *Dgmahl* | Input inventory demand growth moving average half-life |
| *Iufyif* | Input use forecast years into the future |
| *Iitrmt* | Input inventory risk max target |
| *C* | Input inventory target manager base inventory target calculation risk coefficient |
| *Idet* | Input inventory drawdown end time |
| *Idst* | Input inventory drawdown start time |
| *Ibet* | Input inventory build end time |
| *Ibst* | Input inventory build start time |
| *Tidby*[2] | Target increase during build in years |
| *Idob* | Input inventory drawdown over build |

| | |
|---|---|
| *R* | Input inventory risk parameter |
| $w_1$ | Input inventory risk region group weight |
| $w_2$ | Input inventory risk world weight |

1. Separate input values for metal refiners and magnet producers in each region.
2. Separate input values for each REO, RE metal, and region.

#### *2.4.7.2 Product inventory targets*

The inventory target for product *p* is

$$IT_{p,t} = DF_{p,t} \cdot tpiod$$

Where *tpiod* is the target product inventories over demand model input parameter, in years of expected demand. The demand forecast, $DF_{p,t}$, is calculated as a moving average of past demand, with a moving average half-life parameter of $dmahl$.

$$DF_{i,t} = \frac{\sum_{ti=0}^{t}(1 - dmaip)^{ti}\, D_{i,t-1-t}}{\sum_{ti=0}^{t}(1 - dmaip)^{ti}}$$

$$dmaip = 1 - e^{log(0.5)/dma \quad \cdot TS}$$

Product Inventory Target Parameters

| Parameter | Description |
|---|---|
| *Tpiod*[1] | Target product inventory over demand (years) |
| *Dmahl*[2] | Demand moving average half-life |

1. Separate input values for metal refiners and magnet producers in each region.
2. Separate values for producers, mines, illegal oxide suppliers and smugglers.

### 2.4.8 Pricing

#### *2.4.8.1 NdFeB magnet-containing product producer pricing*

Unlike with other agents, NdFeB magnet-containing product producers do not choose prices based on supply and demand, but use a fixed markup of their costs. The price of product $p_t$, is equal to the cost of the magnet plus a fixed cost chosen to make the total cost at the start equal to the starting price. Note that changes in the price of magnets have more of an impact on the price of product if magnet costs get to be a higher share of the cost of the product. Prices are set based on the cost of modeled inputs (oxides) and other costs ($B_p$), with a fixed markup ($m_p$).

$$P_{i,t} = \frac{\sum_s P_{s,i,t} D_{i,s,t}}{\sum_s D_{i,s,t}}$$

$$C_{pc,T,t} = \left(\sum_i n_{pc,T,i} P_{i,t}\right) + B_{pc,T,t}$$

$$P_{pc,t} = m_p\left(\sum_{pl}\left(C_{pc,T(pl),t} \cdot PS_{pc,pl,t}\right)\right)$$

where $T(pl)$ is the technology chosen for product line $pl$, $C_{pc,T,t}$ is the cost of producing using technology $T$, and $PS_{pc,pl,t}$ is the production share for product line $pl$.

#### 2.4.8.2 Illegal oxide producer pricing

Illegal agents set oxide prices for domestic and export sales as a fixed fraction of legal oxide prices. Prices are further adjusted to maintain balance between the export/domestic ratios of illegal supply and demand. The illegal domestic price for each oxide, $P_{ox,il,dom,t}$, is a fixed multiple of the legal domestic price, $P_{ox,l,dom,t}$:

$$P_{ox,il,dom,t} = P_{ox,l,dom,t} \cdot ipdi$$

where *ipdi* is the illegal price discount for internal sales.

The illegal export price for each oxide, $P_{ox,il,exp,t}$, is a fixed multiple of the legal export price with an additional adjustment term if there is an imbalance between the time lag (*t-1*) export shares for demand ($EDR_{t-1}$) and supply ($ES_{t-1}$):

$$P_{ox,il,exp,t} = P_{ox,l,exp,t} \cdot ipde \cdot \exp\big(iodeep \cdot (EDR_{t-1} - ES_{t-1})\big)$$

where *ipde* is the illegal price discount for exports, and $iodeep$ is the illegal oxide demand effect on export prices:

$$EDR_t = \frac{total\ ox.\ \ demand\ from\ rest\ of\ world\ (non\ US\ or\ China)\ buyers}{total\ Chinese\ oxide\ demand}$$

$$ES_t = target\ export\ share\ of\ sales$$

Illegal Oxide Producer Pricing Parameters

| Parameter | Description |
|---|---|
| $ipdi$ | Illegal price discount for internal sales |
| $ipde$ | Illegal price discount for exports |
| $iodeep$ | Illegal oxide demand effect on export prices |

#### 2.4.8.3 Legal mine pricing

Base product prices ($P_{p,t}$) are adjusted based on inventory lags ($I_{p,t-1}$, $I_{p,t-2}$, $I_{p,t-3}$), how inventories are changing over time, and effects from the price response to inventory effects and supply demand effects, along with capacity utilization effects. The following sequence of equations describe how the product price is compiled from a number of different variables and inputs. Note that successive equation definitions of $LP_{p,t} = LP_{p,t} + \cdots$ indicate that the value of the price or log price are updated in sequence, following variable assignment convention in most programming languages.

The new (at time *t*) base product price $P_{p,t}$ depends on the price lag $P_{p,t-1}$ at time *t-1*, the production step markup effect $PSMUE_{p,t}$, the movement to target markup effect, the price response to forecast effect, and the price change forecast lag $PCF_{t-1}$. The log form of the new price is calculated as

$$LP_{p,t} = LP_{p,t-1} + PSMUE_{p,t} + mtmip \cdot (log(tm) - LM_{t-1}) + SDIE_{p,t} + prpf \cdot TS \cdot PCF_{t-1}$$

where $LP_{p,t}$ is the log new price, $LP_{p,t-1}$ is the log price lag, *tm* is the target markup input parameter, $LM_{t-1}$ is the log markup lag, which is calculated by dividing revenues by costs (including operating costs of extraction, amortized capital costs and separation costs) in the previous period, and *prpf* is the price response to price forecast input parameter. The movement to target markup in period, *mtmip* is

$$mtmip = 1 - e^{\frac{log(0.5)}{mtmhl \cdot TS}}$$

Where *mtmhl* is the movement to target markup half-life input parameter.

The supply demand inventory effect $SDIE_{p,t}$ considers the price response to inventory and price response to supply demand. Inventory effects are modeled as

$$IE_{p,t} = \frac{prti}{TS} \cdot log(IR_{p,t})$$

Where *prti* is the price response to inventory, TS is the time step, and $IR_{p,t}$ is the inventory ratio is the ratio of the inventory lag *I* over the inventory target lag *IT*

$$IR_{p,t} = \frac{I_{p,t-1}}{IT_{p,t-1}} \leq 2$$

Supply demand effects consider the price response to last period supply demand input parameter *prlpsd*, the production rate lag *PR*, the demand lag *D*, and the demand expectation lag *DE*

$$SDE_{p,t} = prlpsd \cdot TS \cdot \frac{(PR_{p,t-1} - D_{p,t-1})}{DE_{p,t-1}}$$

The combined $SDIE_{p,t}$ for both supply demand and inventory effects is

$$SDIE_{p,t} = IE_{p,t} + SDE_{p,t}$$

$SDIE_{p,t}$ is constrained between the min and max *SDIE* input parameters:

$$-sdie_{max} \leq SDIE_{p,t} \leq sdie_{max}$$

For inventory lags k > 3, the log price is adjusted by the three previous inventory lags

$$LP_{p,t} = LP_{p,t} - \frac{prcsd}{TS} \cdot \frac{(I_{p,t-1} - 2I_{p,t-2} + I_{p,t-3})}{DE_{p,t-1}}$$

Where $DE_{p,t-1}$ is the demand expectation, *prcsd* is the price response to change in supply and demand input parameter, *TS* is the time step, and $I_{p,t-k}$ are the inventories for lag *k*.

Capacity utilization also influences product prices through the capacity utilization effect

$$CUE_t = (CU_t - cut_{new}) \cdot prcu$$

Where $cut_{new}$ is the new capacity utilization target input parameter when choosing how much new capacity to add, and determines the target maximum capacity utilization during the forecast period, if demand proceeds as forecasted. Lower values mean more new capacity is added. $prcu$ is the price response to capacity utilization input parameter.

Capacity utilization effects are added to the log price

$$LP_{p,t} = LP_{p,t} + CUE_t$$

The new price $P_{p,t}$ is assigned from the price after capacity utilization effect

$$P_{p,t} = e^{LP_{p,t}}$$

Product substitution has a stabilization effect on the new price

$$P_{p,t} = P_{p,t} + SSE_{p,t}$$

The new price is constrained by the price change ratio and the min and max log price, according to the target minimum price $TP_{min}$ and target maximum price $TP_{max}$

$$P_{p,t} = \begin{cases} P_{p,t} + muttmip \cdot (TP_{min} - P_{p,t}), & if\ P_{p,t} < TP_{min} \\ P_{p,t} - mdttmip \cdot (P_{p,t} - TP_{max}), & if\ P_{p,t} > TP_{max} \end{cases}$$

where *muttmip* is the movement up toward target min in period and *mdttmip* is the movement down toward target max in period input parameters.

The new price is further constrained by the log price min and max

$$LP_{p,t,max} = LP_{p,t-1} + log(mpcr)$$

$$LP_{p,t,min} = LP_{p,t-1} - log(mpcr)$$

where *mpcr* is the max price change ratio input parameter. The new price is updated as ether the max value between the new price and the min or the min value between the new price and the max

$$LP_{p,t,min} \leq P_{p,t} \leq LP_{p,t,max}$$

For convenience, the log new price is defined as

$$LP_{p,t} = log(P_{p,t})$$

In the case where $LP_{p,t} < LP_{t-1}$ then $LP_{p,t}$ is adjusted by

$$LP_{p,t} = LP_{p,t} + pdr \cdot (LP_{t-1} - LP_{p,t})$$

Where *pdr* is the price drop reduction input parameter. Further adjustment is made according to the price ratio to regional average

$$LP_{p,t} = LP_{p,t} + mtrapip \cdot \left(-log(BPRTRA_{t-1})\right)$$

The movement to regional average price in period *mtrapip* is

$$mtrapip = 1 - e^{log(1-mtrapim)/(TS/12)}$$

Where *mtrapim* is the movement to regional average price in month, and *TS* is the time steps per year input parameter.

$BPRTRA_{t-1}$ is the buyer price ration to regional average lag.

Finally, the new price is simply

$$P_{p,t} = e^{LP_{p,t}}$$

<u>Export Quota Effects on Price (Mine)</u>

Product prices can be affected by export quotas such that the base product price $P_{p,t}$ is modified to calculate an adjusted price:

$$P'_{p,b,t} = P_{p,b,t} + QPD_{p,b,t}$$

Where $QPD_{p,b,t}$ is the quota price difference at time *t* for buyer *b*:

$$QPD_{p,b,t} = (1 - qrpp) \cdot QPD_{p,b,t-1} + QE_{p,b,t}$$

Where *qrpp* is the quota reduction per period input parameter, $QPD_{p,b,t-1}$ is the quota price difference lag, and $QE_{p,b,t}$ is the current quota effect on product price for product *p*.

Quota effects combine the weighted average log price $\overline{LP}_{p,b,t}$ and demand vs. offer effets:

$$QE_{p,b,t} = e^{\overline{LP}_{p,b,t}} \cdot \left(e^{DVOE_{p,b,t}} - 1\right)$$

The demand vs offer effect $DVOE_{p,t}$ is

$$DVOE_{p,b,t} = mvdoe \cdot e^{log(0.5) \cdot \left(\frac{1-DOO_{p,b,t}}{1-doohe}\right)}$$

Where *mdvoe* is the maximum demand vs. offer effect which is an input parameter that determines how much the demand to offer ratio affects the prices if demand equals offers, and *doohe* is an input parameter that determines how much the demand to offer effect on price changes as the ratio changes. The demand over offer $DOO_{p,t}$ is the ratio of buyer demand lag and quantity offer lag:

$$DOO_{p,t} = min\left(1\ , \quad \frac{D_{p,b,t-1}}{QO_{p,b,t-1}}\right)$$

Mine Pricing Parameters

| Parameter | Description |
|---|---|
| *mtmhl* | Movement to target markup half life input parameter |
| *prpf* | Price response to price forecast |
| *prti* | Price response to inventory |
| *prlpsd* | Price response to last period supply demand |
| $sdie_{max}$ | Max supply demand inventory effect |
| *Prcsd* | Price response to change in supply and demand |
| $cut_{new}$ | New capacity utilization target |
| *prcu* | Price response to capacity utilization |
| *muttmip* | Movement up toward target min in period |
| *mdttmip* | Movement down toward target max in period |
| *mpcr* | Max price change ratio |
| *pdr* | Price drop reduction |
| *mtrapim* | Movement to regional average price in month |
| *qrpp* | Quota reduction per period |
| *mdvoe* | Maximum demand vs. offer effect |
| *doohe* | Demand to offer effect on price change |

### *2.4.8.4 Other agent pricing*

For most products, there is dedicated production capacity used to manufacture each product. Metal refiners and magnet producers are modeled slightly differently: we model agents that have a fixed amount of total capacity, but can choose to use that capacity to produce different kinds of magnets or refine different rare earth metals. This affects the pricing of different metals/magnets, since differences in profitability should not persist when the agent can switch to producing the more profitable product. To capture this, some calculations are done at the production capacity level, indexed by pc, where there may be either one or more products (p) associated with each production capacity (pc). $P_{pc}$ is the set of products produced with a given production capacity, which for most producers will be a single product, except in the case of metal refiners and magnet producers, in which case all products they can produce are included.

$$ln(AM_{pc,t}) = \ln(AM_{pc,t-1}) + IE_{pc,t} + SDE_{pc,t} + CUE_{pc,t} + TME_{pc,t}$$

Where $AM_{pc,t}$ is the average markup of products that can be produced with a given production capacity. $AM_{pc,t-1}$ is calculated using $S_{p,t-1} * IC_{p,t-1}$ as weights, where $S_{p,t-1}$ is sales in the last period, and $IC$ is input cost per unit sold.

The inventory effect $IE_{p,t}$ is modeled as

$$IE_{p,t} = \frac{prti}{TS} \cdot log\left(IR_{p,t}\right)$$

Where *prti* is the price response to inventory, TS is the time step, and $IR_{p,t}$ is the inventory ratio is the ratio of the inventory lag *I* over the inventory target lag *IT*

$$IR_{p,t} = \frac{I_{p,t-1}}{IT_{p,t-1}}$$

Supply demand effects consider the price response to last period supply demand input parameter *prlpsd*, the production rate lag *PR*, the demand lag *D*, and the demand expectation lag *DE*

$$SDE_{p,t} = prlpsd \cdot TS \cdot \frac{\left(PR_{p,t-1} - D_{p,t-1}\right)}{DE_{p,t-1}}$$

The supply demand and inventory effects are each constrained by minimum and/or maximum values:

$$IE_{p,t} \leq \ln(ie_{max})$$

$$-\ln(sde_{max}) \leq SDE_{p,t} \leq \ln(sde_{max})$$

Capacity utilization also influences prices. The capacity utilization effect is defined as:

$$CUE_t = (CU_t - cut_{new}) \cdot prcu$$

Where $cut_{new}$ is the new capacity utilization target input parameter when choosing how much new capacity to add, and determines the target maximum capacity utilization during the forecast period, if demand proceeds as forecast. Lower values mean more new capacity is added. $prcu$ is the price response to the capacity utilization input parameter.

The target markup effect is calculated as

$$TME_{pc,t} = mtmip \cdot (\ln(TAM_{pc,t-1}) - \ln(AM_{pc,t-1}))$$

The movement to target markup in period, *mtmip* is

$$mtmip = 1 - e^{\left(\frac{log(0.5)}{mtmhl \cdot TS}\right)}$$

Where *mtmhl* is the movement to target markup half-life input parameter.

Average target markups, like average markups, are calculated using $S_{p,t-1} * IC_{p,t-1}$ as weights.

Average input costs, $IC_{p,t}$, of producing the product is calculated, including the costs of purchasing modeled products as well as other production costs.

The profit per unit ($\pi$) is calculated as

$$\pi_{pc,t} = \frac{\sum_{p \in P_{pc}} S_{p,t-1} IC_{p,t} \cdot \ (AM_{pc,t} - 1)}{\sum_{p \in P_{pc}} S_{p,t-1}}$$

Where $CIC = \sum_{p \in P_{pc}} S_p IC_p$ is combined input cost, $CS = \sum_{p \in P_{pc}} S_p$ is combined sales. $\pi$ is the same for all products produced using the same production capacity, since if they were not, the producer could switch to producing more of the more profitable product. The value for $\pi$ is chosen to be the solution to the equation:

$$\frac{\sum_{p \in P_{pc}} S_{p,t-1} \cdot (IC_{p,t} + \pi_{pc,t})}{\sum_{p \in P_{pc}} S_{p,t-1} \cdot IC_{p,t}} = AM_{pc,t}$$

so that the combined markup for all products produced using the same production capacity is $AM_{pc,t}$.

In addition to the input costs and the profit, the base price ($P$) for each product and buyer also includes an adjustment so that the prices an agent's buyers will have to pay is not too different from the regional average:

$$\ln\left(P_{p,b,t}\right) = \ln\left(IC_{p,t} + \pi_{pc,t}\right) + RAE_{p,b,t}$$

$$RAE_{p,b,t} = mtrapip \cdot \left(\ln\left(RABP_{p,b,t-1}\right) - \ln(ABP_{p,b,t-1})\right)$$

Where RABP is the regional average buyer price and ABP is the average buyer price, where buyer prices include transaction costs (export taxes, tariffs and the cost of insurance and freight), as described in the 'Transaction costs' section. The movement to regional average price in period *mtrapip* is

$$mtrapip = 1 - e^{log(1-mtrapim)/(TS/12)}$$

Where *mtrapim* is the movement to regional average price in month, and *TS* is the time steps per year input parameter.

Export Quota Effects on Price

Product prices can be affected by export quotas such that the base product price $P_{p,t}$ is modified to calculate an adjusted price:

$$P'_{p,b,t} = P_{p,b,t} + QPD_{p,b,t}$$

Where $QPD_{p,b,t}$ is the quota price difference at time *t* for buyer *b*:

$$QPD_{p,b,t} = (1 - qrpp) \cdot QPD_{p,b,t-1} + QE_{p,b,t}$$

Where *qrpp* is the quota reduction per period input parameter, $QPD_{p,b,t-1}$ is the quota price difference lag, and $QE_{p,b,t}$ is the current quota effect on product price for product *p*.

Quota effects combine the weighted average log price $\overline{LP}_{p,b,t}$ and demand vs. offer effets:

$$QE_{p,b,t} = e^{\overline{LP}_{p,b,t}} \cdot \left(e^{DVOE_{p,b,t}} - 1\right)$$

The demand vs offer effect $DVOE_{p,t}$ is

$$DVOE_{p,b,t} = mvdoe \cdot e^{log(0.5) \cdot \left(\frac{1-DOO_{p,b,t}}{1-doohe}\right)}$$

Where *mdvoe* is the maximum demand vs. offer effect which is an input parameter that determines how much the demand to offer ratio affects the prices if demand equals offers, and *doohe* is an input parameter that determines how much the demand to offer effect on price changes as the ratio changes. The demand over offer $DOO_{p,t}$ is the ratio of buyer demand lag and quantity offer lag:

$$DOO_{p,t} = min\left(1\,, \quad \frac{D_{p,b,t-1}}{QO_{p,b,t-1}}\right)$$

Other Agent Pricing Parameters

| Parameter | Description |
|---|---|
| *prti* | Price response to inventory |
| *prlpsd* | Last period supply demand |
| $cut_{new}$ | New capacity utilization target input parameter when choosing how much new capacity to add |
| $prcu$ | Price response to capacity utilization |
| *mtmhl* | Movement to target markup half-life |
| *mtrapim* | Movement toward regional average price in month |
| *TS* | Time steps per year |
| *qrpp* | Quota reduction per period |
| $sde_{max}$ | Supply demand effect max |
| $ie_{max}$ | Inventory effect max |
| *mdvoe* | Maximum demand vs. offer effect |
| *doohe* | Demand to offer effect on price change |

### 2.4.9 Buyers – export amounts

#### *2.4.9.1 Illegal sales targets*

The total sales targets for each oxide ($TST_{ox,t}$) are based on production ($PR_{ox,t}$) and inventories ($I_{ox,t}$), relative to target product inventory levels ($IT_{ox,t}$) from the ‘Product inventory target’ section:

$$TST_{ox,t} = PR_{ox,t} + seias \cdot \left(I_{ox,t} - IT_{ox,t}\right)$$

Where *seias* is the share of excess inventories in available supplies input parameter.

Total sales targets are distributed to buyers by first dividing into export targets and domestic sales targets. Illegal exports require additional smuggling risks, so for illegal exports to occur, price differential between exports and domestic sales must justify the added risk. The share of illegal production targeted for exports depends on the price differential between export prices and internal prices, relative to smuggling costs. Smuggling costs of each rare earth are assumed to be distributed following a log-normal distribution. Note that this is an aggregate cost function, which represents the fact that while some producers might be able to export at low cost, others would find it more costly.

The scale and shape parameters are assumed to be the same for all oxides, and are chosen so that the total export share of all oxides at 2010 price differential is determined by the input parameter, *ieepd*, while the elasticity of total export share in response to a small change in the price differential is determined by the input parameter, *ieespde*.

The cumulative distribution function for the log-normal distribution is used to calculate the equilibrium share for a given price differential:

$$ES\_Equ_{ox,t} = CumDist(priceDifferential_{ox,t})$$

The export share ($ES_{ox,t}$) in each period then moves part way to the new equilibrium:

$$ES_{ox,t} = ES_{ox,t-1} + meeip \cdot (ES\_Equ_{ox,t} - ES_{ox,t-1})$$

The movement to equilibrium exports in period, *meeip* is

$$meeip = 1 - e^{\frac{log(0.5)}{meehl \cdot TS}}$$

Where *meehl* is the movement to equilibrium exports half-life input parameter.

Once the export share, $ES_{ox,t}$, is determined, the domestic and export portions of sales targets are distributed to individual buyers (b) who are willing to buy illegally in proportion to their total input request ($TD_{b,t-1}$) in the previous period:

$$ST_{ox,\,b,t} = \frac{TD_{ox,b,t-1}}{\sum_{b2 \in DB} TD_{ox,b2,t-1}} \cdot TST_{ox,t} \cdot (1 - ES_{ox,t}) \qquad \text{for buyers } b \in DB\text{, the set of domestic buyers}$$

$$ST_{ox,b,t} = \frac{TD_{ox,b,t-1}}{\sum_{b2 \in EB} TD_{ox,b2,t-1}} \cdot TST_{ox,t} \cdot ES_{ox,t} \qquad \text{for buyers } b \in EB\text{, the set of export buyers}$$

US buyers are not willing to buy illegal oxides, though they may buy downstream products produced using illegal oxides.

Illegal Sales Target Parameters

| Parameter | Description |
|---|---|
| *ieepd* | Illegal exporter export share at 2010 price differential |
| *ieespde* | Illegal exporter export share price differential elasticity |
| *meehl* | Movement to equilibrium exports half-life |
| *seias* | Share of excess inventories in available supplies |

#### *2.4.9.2 Quantity offers for legal agents*

Suppliers don't have full control who their buyers are, but they do choose a maximum offer to make to each buyer, which is intended to offer enough to each potential buyer to meet expected demand if possible, while not exceeding the amount they have available to sell, or sales limits based on export quotas. If quotas forces sales to be lower than demand, they prioritize who to sell to and which products to sell based on which sales would be most profitable.

First, the offer before reduction, $OBR_{p,b,t}$ is calculated by allocating the maximum offer to different buyers based on several buyer characteristics intended to capture potential demand: the amount requested by each buyer in the previous period (D), each buyer's total demand from all suppliers (TD), any additional demand that the buyer could have if its production weren't limited (ENP). A small amount is also allocated evenly to all buyers.

$$OBR_{p,b,t} = AABD_{p,t} \cdot \frac{D_{p,b,t-1}}{\sum_b D_{p,b,t-1}} + AATD_{p,t} \cdot \frac{TD_{p,b,t-1}}{\sum_b TD_{p,b,t-1}} + AAENP_{p,t} \cdot \frac{ENP_{p,b,t-1}}{\sum_b ENP_{p,b,t-1}} + AAE_{p,t} \cdot \frac{1}{|B|}$$

where $TD_{p,b,t-1}$ is the total buyer demand (from all suppliers) lag, $ENP_{p,b,t-1}$ is the extra new potential demand lag, and $D_{p,b,t-1}$ is the buyer demand (from supplier) lag.

The amount to allocate by buyer demand ($AABD_{p,t}$) is set to:

$$AABD_{p,t} = \sum_b D_{p,b,t-1} \cdot btood + TSR_{p,t} \cdot btoost$$

where $TSR_{p,t}$ is the target sales rate, *btood* is the base target over offer demand input parameter, and *btoost* is the base target offer over sales target input parameter.

$AATD_{p,t}$ is the amount to allocate by total demand:

$$AATD_{p,t} = TSR_{p,t} \cdot eoost$$

where *eoost* is the extra offer over sales target input parameter.

$AAENP_{p,t}$ is the amount to allocate by extra new potential:

$$AAENP_{p,t} = \sum_b ENP_{p,b,t-1} \cdot eoonpd$$

where *eoonpd* is the extra offer over new potential demand input parameter.

$AAE_{p,t}$ is the amount to allocate evenly:

$$AAE_{p,t} = MSR_{p,t} \cdot mooms$$

where $MSR_{p,t}$ is the max sales rate, and *mooms* is the min offer over max sales input parameter.

Each of these is scaled down by the same multiplier, if needed, so that the total amount to allocate does not exceed the maximum sales rate:

$$mult = \min(\frac{MSR_{p,t}}{AABD_{p,t} + AATD_{p,t} + AAENP_{p,t} + AAE_{p,t}}, 1.0)$$

$$AABD_{p,t} = AABD_{p,t} \cdot mult$$

$$AATD_{p,t} = AATD_{p,t} \cdot mult$$

$$AAENP_{p,t} = AAENP_{p,t} \cdot mult$$

$$AAE_{p,t} = AAE_{p,t} \cdot mult$$

These offers before reduction (OBR) are then reduced if needed based on export limits (ESL), which are calculated in the 'Government export quotas' section.

Long term contracts get priority – if there are limited supplies available, long term contracts are met first if possible. If not all long term contracts can be met, long term offers ($LTO_{p,b,t}$) are all reduced from the contract amounts by the same percentage.

If the long term plus short term offers exceed the amount of the export quotas, then the amount that needs to be reduced is calculated as the total short term reduction TSTR:

$$TSTR = min(\sum_{p,b} OBR_{p,b,t} - ESL_t, \quad \sum_{p,b} OBR_{p,b,t} - \sum_{p,b} LTD_{p,b,t})$$

where $ESL_t$ is the amount of the export sales limit for the agent in period, as calculated in the 'Government export quotas' section, and $LTD_{p,b,t}$ is demand from long-term contracts.

If they need to reduce short-term sales below their targets based on demand, they prioritize sales that would provide the most benefit from selling to the specified buyer relative to the world average, so they reduce offers more for buyers with lower price differentials:

$$PD_{p,b,t} = \frac{P_{p,b,t} - AP_{p,t}}{npd}$$

The average price is:

$$AP_{p,t} = \frac{\sum_b P_{p,b,t} OBR_{p,b,t}}{\sum_b OBR_{p,b,t}}$$

The short term reduction is:

$$STR_{p,b,t} = TSTR \cdot \frac{OBR_{p,b,t} \cdot e^{-PD_{p,b,t}}}{\sum_{b,p} OBR_{p,b} \cdot e^{-PD_{p,b,t}}}$$

The short term offer is:

$$STO_{p,b,t} = OBR_{p,b,t} - LTD_{p,b,t} - SRT_{p,b,t}$$

The total amount offered to each buyer, $QO_{p,b,t} = STO_{p,b,t} + LTO_{p,b,t}$

Quantity Offers for Legal Agents Parameters

| Parameter | Description |
|---|---|
| $btood$ | Base target offer over demand |
| $btoost$ | Base target offer over sales target |
| $eoost$ | Extra offer over sales target |

| | |
|---|---|
| $eoonpd$ | Extra offer over new potential demand |
| $mooms$ | Min offer over max sales |
| $npd$ | Normal price differential |

2.4.10 Input purchases

Agent request enough inputs to meet their production and move inventories in the direction of inventory targets.

$$D_{p,t} = \frac{iimttip}{TS} \cdot \left(TI_{p,t} - I_{p,t-1}\right) + IDP_{p,t}$$

with the restriction that it must be at least as high as their contracted purchasing amount. The input parameter $iimttip$ is the input inventory movement towards target in period, and $TS$ is the time step.

The inventory desired for production ($IDP_{p,t}$) depends on production rates ($PR_{p,t-1}$), and on the technology relationships $n_{p,i}$, the amount of input $i$ needed to produce one unit of product $p$.

$$IDP_{p,t} = \sum_{p2} n_{p2,p} \cdot PR_{p2,t-1}$$

Input Purchasing Parameters

| Parameter | Description |
|---|---|
| $Iimttip$ | Input inventory movement towards target in period |
| $TS$ | Time steps per year |

2.4.11 Price response

Final demand agents respond to product prices modeled by

$$D_t = (1-\delta) \cdot D_{t-1} \cdot \left(\frac{P_t}{P_{t-1}}\right)^{\varepsilon_{im}} + \delta \cdot \overline{D}_t \cdot \left(\frac{P_t}{\overline{P}}\right)^{\varepsilon_{lr}}$$

Where:

$\varepsilon_{lr}$ = long run elasticity, taken from the input data
$\varepsilon_{im} = ipsre \cdot \varepsilon_{sr}$ is the immediate elasticity, $\varepsilon_{sr}$ is the short-run elasticity from the input data, which includes the immediate elasticity as well as the first year of the movement toward the long-run elasticity
$\delta = \left(1 - \frac{\varepsilon_{sr}}{\varepsilon_{lr}}(1 - ipsre)\right)^{TS}$ = rate of convergence to long run equilibrium in a period
$ipsre$ is the immediate portion of short-run elasticity input parameter.
$D_t$ = demand at time *t*
$P_t$ = price at time *t*
$\overline{P}$ = the baseline price, defined as the value used to formulate the demand projections
$\overline{D}_t$ = long-run demand at time *t* and price $\bar{p}$ [input values defined by scenario]

Price Response Parameters

| Parameter | Description |
|---|---|
| $ipsre$ | Immediate portion of short-run elasticity |

### 2.4.12 Suppliers – import amounts

All agents that make purchases, including final demand agents and all producers except mines and Illegal oxide suppliers, first choose how much to buy legally (if relevant), and then choose among legal suppliers based on their purchases the previous period and the relative prices of their suppliers.

Illegal Purchases

Agents who buy illegal oxides determine their illegal share of purchases ($IS_{p,t}$) by moving toward a target ($IST_{p,t}$) based on the total of the targets set by the illegal suppliers, $ST_{p,t} = \sum_s ST_{s,p,t}$, and total demand (legal and illegal) by the buyer, $TD_{p,t}$

$$IST_{p,t} = min\left(\frac{ST_{p,t}}{TD_{p,t}},\ 1\right)$$

$$IS_{p,t} = IS_{p,t-1} + ismtp \cdot \left(IST_{p,t} - IS_{p,t-1}\right)$$

where $ismtp$ determines the illegal share of purchases movement toward target in period.

The share of legal demand that is requested from each supplier, $LSS_{i,s,t}$, adjusts gradually from its previous value based on the difference between prices in the two regions following:

$$LSS_{p,s,t} = LSS_{p,s,t-1} - TS \cdot isrpd \sum_{\bar{s} \neq s} ln\left(\frac{P_{s,p,t}}{P_{\bar{s},p,t}}\right)$$

This is subject to the restriction that supplier shares, $LSS_{i,s,t}$, must be between 0 and 1. $isrpd$ is the input share response to price difference, $TS$ is the time step.

The share of illegal demand that goes to each supplier, $ISS_{p,s,t}$, is in proportion to illegal sales targets, $ST_{s,b,t}$ of the different illegal suppliers.

In total, the supplier shares for individual legal suppliers are $SS_{p,s,t} = (1 - IS_{p,t}) \cdot LSS_{p,s,t}$, and for individual illegal suppliers are $SS_{p,s,t} = IS_{p,t} \cdot ISS_{p,s,t}$

Short term demand (total demand, $D_{i,t}$, as calculated in input purchases or price response sections, minus demand from long-term contracts, $LTD_{i,t}$) is apportioned to suppliers based on these supplier shares:

$$D_{i,s,t} = LTD_{i,s,t} + (D_{i,t} - LTD_{i,t}) \cdot SS_{i,s,t}$$

Supplier Choice Parameters

| Parameter | Description |
|---|---|
| $Ismtp$ | Illegal share movement towards production |
| $Isrpd$ | Input share response to price difference |
| $TS$ | Time steps per year |

### 2.4.13 Production rate

#### *2.4.13.1 Common production rule*

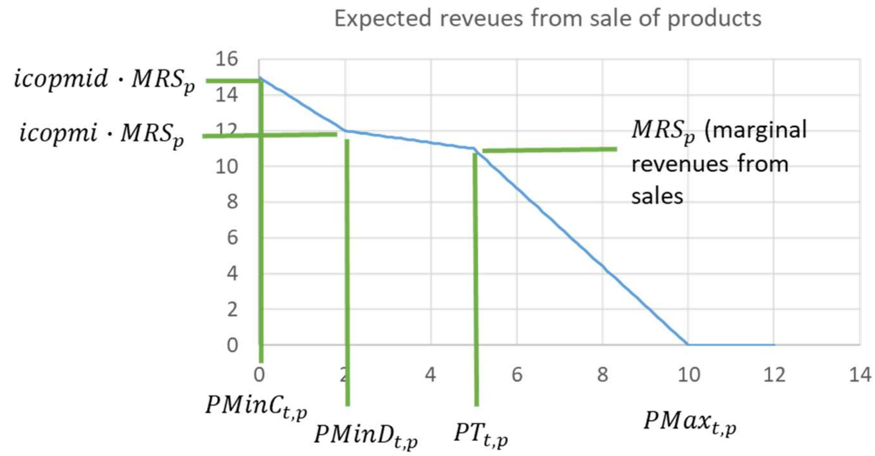
Agents choose how much to produce, $PR_T$, with each technology $T$ by maximizing expected profits. This is done by finding the production rate at which expected marginal revenues from the sale of products equals the expected cost of purchasing inputs and producing the product, subject to a production capacity limit.

<u>Marginal Revenues</u>

$ERS_{T,p}$ is the expected marginal benefit from increasing production of product *p* using a technology *T*. When production is at the desired level to meet demand (with an inventory adjustment), the marginal benefit of producing is equal to the marginal revenues received from sales of the product. Producing more than is needed to meet demand will have decreasing marginal benefits, as prices may need to drop to be able to sell all of the product being produced. Producing less than is needed to meet demand will drop inventories below target levels. If production isn't enough to meet demand even after depleting inventories, the costs are higher, as this may hurt their ability to keep their customers. At a minimum, producers will produce enough to meet contracts, if possible.

To capture this, the marginal revenue function is a decreasing piecewise linear function formed by connecting the following points with linear segments:

| **Amount produced** | **Marginal revenues** |
|---|---|
| $PMinC_{T,p}$, the minimum production needed to meet contracts | $icopmid \cdot MRS_p$ |
| $PMinD_{T,p}$, the minimum production needed to meet demand | $icopmi \cdot MRS_p$ |
| $PT_{T,p}$, the target production that would meet demand with target inventories | $MRS_p$ |
| $PMax_{T,p}$, the maximum useful production | 0 |




Where:

$PMinC_{T,p} = \left(CN_p - piop \cdot \frac{I_p}{TS}\right)/tp_{T,p}$ is the minimum production with contracts

$PMinD_{T,p} = \left(D_p - piop \cdot \frac{I_p}{TS}\right)/tp_{T,p}$ is the minimum production with demand

$TP_{T,p} = (D_p + \frac{IT_p - I_p}{TS} \cdot seias)/tp_{T,p}$ is the target production based on adjusting demand to account for inventories

$PMax_{T,p} = (D_p + \frac{IT_p \cdot mii - I_p}{TS} \cdot mimum)/tp_{T,p}$ is the maximum useful production for product $p$, for technology $T$

$CN_p$ are contract needs for product p, and $D_p$ are demands for product p, $IT_p$ are inventory targets, $I_p$ are inventories. $tp_{T,p}$ are the amounts produced of each product per unit of production with technology T. $MRS_p = SP_p \cdot tp_{T,p}$ are the marginal revenues from sales, based on prices the producer can get from selling a product, $SP_p$. $TS$ is the time step size.

<u>Marginal Costs</u>

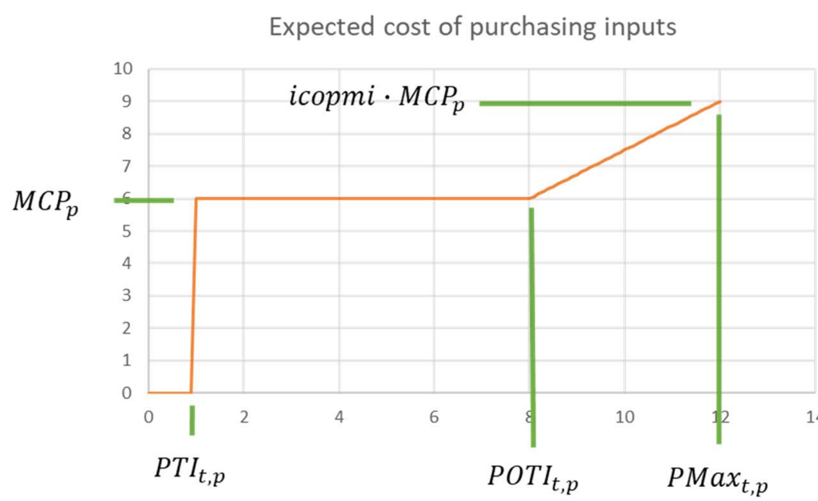
$$MC_{T,p} = MCPI_{T,p} + MCMI_{T,p} + MTC_T$$

$MCPI_{T,p}$ is the expected marginal cost from use of purchased input $p$ due to increasing production of technology $T$. $MCMI_{T,p}$ is the expected marginal cost from use of manufactured input $p$, which only applies to decisions of how much to separate mine products, where the inputs to the separation step is also generated by the same Mine/Separation agent. $MTC_T$ is the marginal technology costs, which covers all other costs of production not explicitly included in the model, including purchases of products that aren't modeled.

The marginal cost of purchased inputs, $MCPI_{T,p}$, is generally based on the purchase costs of inputs, if enough inputs are available for purchase. If input needs are greater than what is available, the costs of production increase to a max level of drawdown from inventories. Production beyond that point is not allowed. If input inventories are higher than targets, then there is assumed to be no cost of using enough inventories to bring input inventories toward targets at the desired rate.

To capture this, the marginal cost function is an increasing piecewise linear function formed by connecting the following points with linear segments:

| Amount produced | Marginal costs |
|---|---|
| $< PTI_{T,p}$, production level to bring inventories toward targets with no purchases | 0 |
| $PTI_{T,p}$ | $MCP_p$ |
| $POTI_{T,p}$, production level to bring inventories toward targets with max purchases | $MCP_p$ |
| $PMax_{T,p}$, maximum possible production with max purchases and inventory use | $icopmi \cdot MCP_p$ |




Where:

| | |
|---|---|
| $MCP_{T,p} = BP_p \cdot ti_{T,p}$ | is the marginal cost due to purchases of input $p$: |
| $ti_{T,p}$ | are the amounts used of each input per unit of production with technology $t$ |
| $BP_p$ | are the buyer prices the producer would pay to buy a product $p$. |
| $QO_p$ | is the total quantity offer received from all suppliers |
| $PTI_p = \left(\frac{I_p - IT_p}{TS} \cdot seias\right) / ti_{T,p}$ | is production to use target inventories |
| $POTI_{T,p} = \left(QO_p + \frac{I_p - IT_p}{TS} \cdot seias\right) / ti_{T,p}$ | is production to use offers and target inventories |
| $PMax_{T,p} = \left(QO_p + piop \cdot \frac{I_p}{TS}\right) / ti_{T,p}$ | is the maximum production |

The marginal cost of manufactured inputs, $MCMI_{T,p}$, is the inverse of the marginal revenue function, since any inputs not used in the production step will be available for sale and subject to the same marginal revenue function. The amount available for sale will be $UP_p - PRT_T \cdot ti_{T,p}$, where $UP_p$ is upstream production of product p, $PRT_T$ is the production rate target of technology T (which is being adjusted to maximize profits) and $ti_{T,p}$ are the amounts used of each input per unit of production with technology *T*.

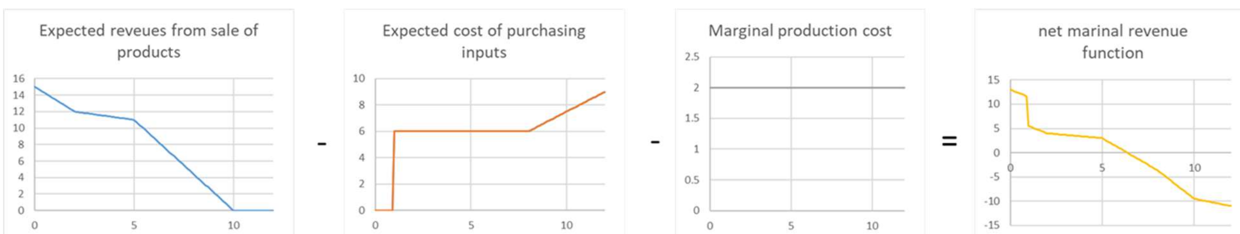
Marginal technology costs, $MTC_T$, are a assumed to be constant and are specified in the input data.

<u>Capacity Limit</u>

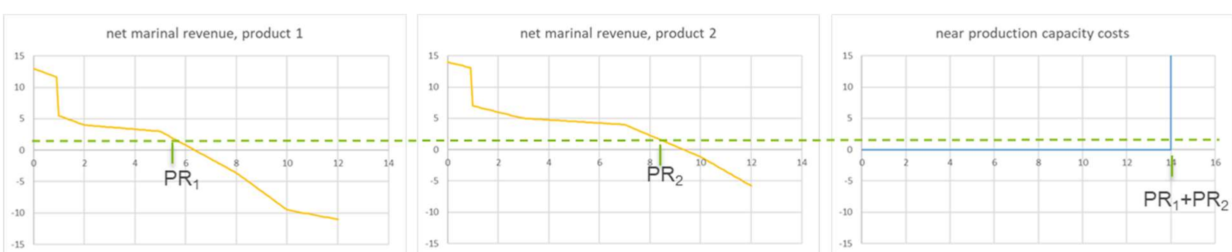
Total capacity for a given technology cannot exceed a maximum production capacity, $PC_{PC}$.

<u>Net Marginal Revenue</u>

The marginal revenue and cost functions are combined (revenues – costs) into a single net marginal revenue function associated with each technology:




If the production capacity can be used to produce one product, then the production level is found so that the net marginal revenue equals the near-production-capacity cost. If there is more than one possible product that can be produced using the same production capacity, as with metals, then a point is found where the marginal revenues (mr) for producing each product are all equal, and are also equal to the marginal costs (mc) of being near capacity at that total production level (or are at the maximum production capacity if marginal revenues > costs at maximum), as illustrated below:



The result of this optimization is a target production level ($PRT_{T,t}$) for each technology. Actual production levels, $PR_{T,t}$ move a portion of the way to target production levels in each period, with the speed of convergence related to how much changing to the new target production level would increase profits.

$$PR_{T,t} = PR_{T,t-1} + MC \cdot PMC$$

$$MC = (PRT_{T,t} - PR_{T,t-1}) \cdot mmnt$$

$$PMC = 1 - (1 - pom1)^{PIR}$$

$$PIR = \frac{PI \cdot PR_{MA}}{MC \cdot TC_{MA}}$$

Where:

$MC$ is the maximum change

$PMC$ is the portion of max change to move

$PIR$ is the profit improvement rate,

$PI$ is the profit improvement from switching from $P_{T,t-1}$ to $P_{T,t-1} + MC$

$PR_{MA}$ is a moving average of past production levels

$TC_{MA}$ is a moving average of past total costs of production

Initial Production Capacity Influence on Production Rates

Finally, production is not allowed to exceed a minimum capacity utilization, and it cannot exceed capacity. The actual production rates $PR_{p,t}$ are adjusted if needed according to available production capacity, $PC_{p,t}$

$$PR_{p,t} = \begin{cases} PC_{p,t} \ if \ PR > PC \\ PC_{p,t} \cdot cu_{p,min} \ if \ PR < PC \cdot cu_{p,min} \\ PR_{p,t} \ otherwise \end{cases}$$

where $cu_{p,min}$ is the minimum capacity utilization input parameter.

Common Production Rule Parameters

| Parameter | Description |
|---|---|
| *seias*[1] | Share of excess inventories in available supplies |
| *Piop*[2] | Portion of inventory to offer in period |
| *mii* | Maximum inventories over inventories |
| $mimum$ | Maximum inventory movement to useful max |
| $icopmi$ | Input cost over price min inventory |
| $icopmid$ | Input cost over price min inventory and demand |

| | |
|---|---|
| *TS* | Time steps per year |
| *Mmnt*[3] | Max movement to new target |
| *pom1* | Portion of max if profit improvement rate is 1 |
| $cu_{p,min}$ | Minimum capacity utilization |

1. Separate values for metal refiners, mine, and illegal oxide suppliers.
2. Separate values for mine, and illegal oxide suppliers, and general producers.
3. Separate values for mines.

*2.4.13.2 Chinese production quotas*

Production from Chinese mines are limited by production quotas. Total production quotas are defined in the input data, and quotas are distributed to the different Chinese mines in proportion to their production capacity. Chinese mines decide how much to produce using the same common production rule as other agents, but with the maximum production capacity reduced to match the production quota.

*2.4.13.3 Oxide separation*

Oxide separation is done by mining and illegal oxide supplier agent. We model oxide production in the following steps:

1. Oxide is extracted and a mixed oxide is produced containing Ce, La, Nd and Pr, while heavy and SEG rare earths are separated into Dy, Tb, Eu, Gd, Sm, and Y oxides.
2. A portion of the CeLaNdPr mixed oxide is separated into CeLa and NdPr oxides.
3. A portion of the CeLa oxide is separated into Ce and La oxides
4. A portion of the NdPr oxide is separated into Nd and Pr oxides.

The result is that a mix of Dy, Tb, Eu, Gd, Sm, Y, Ce, La, Nd, Pr, CeLa, Didymium (NdPr) and mixed (CeLaNdPr) oxides.

The amount that is extracted and the amount that is separated at each phase is determined using the production rule described in the 'Common production rule' section.

*2.4.13.4 Illegal production*

Each illegal agent produces rare earths in fixed proportions, which are specified as $PS_{ox}$, the share of total rare earth oxide (TREO) in the deposit that is of oxide $ox$.

Target extraction rates for TREO, $PRT_{TREO,t}$ are set using the common production rule described in the 'Common production rule' section, and separation rates are set as described in the 'Oxide separation' section. Demand used as an input to the production rule is calculated differently from other types of agents. Total illegal demand (TID) is based on a separate calculation of domestic (China) illegal demand (DID) and foreign (non-China) illegal demand (FID). Domestic illegal demand is the amount of domestic demand (DD) in excess of domestic legal production (DLP). Foreign illegal demand (FID) is calculated as foreign demand (FD) in excess of foreign production (FP) minus legal export demand (LED), which is a share of export demand based on the amount legal production that is available relative to total demand. Legal exports are also subject to a limit based on export quotas

(EQ). Restrictions on which buyers are allowed to buy illegally are also accounted for. This total illegal demand is then assigned to individual agents in proportion to their production in the previous period.

$$TID = DID + FID$$

$$DID = \max(0, DD - DLP)$$

$$FID = \max(0, FD - FP - LED)$$

$$LED = \min\left(FD - FP, (FD - FP) \cdot \frac{DLP}{DD + FD - FP}, EQ\right)$$

The Chinese government may try to limit illegal production, but sources differ on how much effort they will make on this goal. We model this by specifying government targets for illegal supply, $GT_t$, that may change over time and can depend on the scenario used. The government target is assigned to individual agents in proportion to their target production. If the government target is less than the supplier target ($PRT_{TREO,t}$) then production ($PR_{TREO,t}$) is reduced:

$$PR_{TREO,t} = \min(PRT_{TREO,t}, PRT_{TREO,t} - mtgpt \cdot (PRT_{TREO,t} - GT_{TREO,t}))$$

Where $mtgpt$ is movement toward government production target.

Illegal Production Parameters

| Parameter | Description |
|---|---|
| $mtgpt$ | Movement toward government production target |

#### *2.4.13.5 NdFeB magnet production*

Total magnet production is calculated using the common production rule described in the 'Common production rule' section, using average prices, input use and production costs for the different types of magnet in the proportions produced in the previous period.

Production targets for each magnet type, $PT_{p,t}$, are divided in order to make inventories of the different magnet types proportional to target inventory levels when possible, given the restriction that production of each magnet type must be non-negative.

#### *2.4.13.6 NdFeB magnet-containing product production*

Production decisions are modeled more simply for NdFeB magnet containing product producers. Total production rates $PR_{pc,t}$ are set to be enough to match demand. Production levels for each product line are determined by production shares. Production levels for newly initiated product lines are equal to the average production level of all lines in production,

$$PR_{pc,pl,t} = PR_{pc,t} \cdot PS_{pc,pl,t}$$

Production levels for the different product lines adjust over time in the direction of equilibrium production shares that depend on the relative costs of the production for each product line (based on different technologies used).

$$PSEq_{pc,\overline{pl},t} = \frac{\left(\frac{1}{C_{pc,T(\overline{pl}),t}}\right)^{eupsercd}}{\sum_{pl}\left(\frac{1}{C_{pc,T(pl),t}}\right)^{eupsercd}}$$

$$PS_{pc,pl,t} = PS_{pc,pl,t-1} + eupsecip\left(PSEq_{pc,pl,t} - PS_{pc,pl,t-1}\right)$$

Where $C_{pc,T(pl),t}$ is the cost of producing using the technology $T$ being used for the product line $pl$.

$$eupsecip = 1 - e^{\frac{log(0.5)}{eupsechl \cdot TS}}$$

*eupsechl* is the end use production share equilibrium convergence half-life input parameter, and *eupsercd is the* end use production share equilibrium response to cost difference input parameter.

NdFeB Magnet-containing Product Production Parameters

| Parameter | Description |
|---|---|
| *Eupsechl* | End use production share equilibrium convergence half-life |
| *Eupsercd* | End use production share equilibrium response to cost diff. |

### 2.4.14 Production, sales and inventory updating

#### *2.4.14.1 Production, inventory updating*

Production occurs at the production rate calculated in the previous period, as long as sufficient inventories are available for the production to occur. This adds to product inventories ($I_{p,t}$) and subtracts from input inventories ($I_{i,t}$).

$$I_{p,t} = I_{p,t-1} + TS \cdot PR_{p,t}$$

$$I_{i,t} = I_{i,t-1} - TS \cdot \sum_{p} n_{p,i} \cdot PR_{p,t}$$

where $TS$ is the time step (which converts annualized flow data into a per period change in stock), and $n_{p,i}$ is the amount of input $i$ needed to produce one unit of product $p$.

For mines, the oxides they extract are removed from resources, rather than inventories.

#### *2.4.14.2 Sales, inventory updating*

Agents will sell as much, $S_{p,s,b,t}$, as is requested from its buyers, $D_{p,s,b,t}$ plus long-term contract amounts, up to a maximum of the amount offered, $QO_{p,s,b,t}$. Long term contracts are given preference of short term requests. Inventories of both the supplier and buyer are updated when sales occur.

Supplier inventories for supplier $s$:

$$I_{p,s,t} = I_{p,s,t-1} - TS \cdot \sum_{b} S_{p,s,b,t}$$

Buyer inventories for buyer $b$:

$$I_{p,b,t} = I_{p,b,t-1} + TS \cdot \sum_{s} S_{p,s,b,t}$$

## 2.5 Model Initialization

### 2.5.1 Initializing Demand

A baseline demand level is specified for all final products and each time step of the model run. These are used to initialize final demand, and as a starting point for subsequent demand calculations throughout the model run. Baseline final product demands are calculated for each final product based on the demand scenarios and technology relationships from the input data. The demand scenarios specify the amount of each REO used in each final product at the beginning of each year from 2010 to 2030. Values throughout the model run are estimated from these values through linear interpolation between the two nearest points. Technology relationships for each production process specify the baseline amount of inputs needed to produce a product; for example, the amount of Nd metal, Dy metal, Di metal and Pr metal needed to produce a magnet for industrial motors. The demand for final products is calculated so that if the rare earth oxide needs based on these technology relationships are traced back, they will match the oxides needs specified in the demand scenario as closely as possible. If there is an inconsistency between the technology relationships and the ratios of oxides from the demand scenario, the technology relationships are used, and the final demand values are chosen to minimize the sum of the squared differences between the calculated oxide needs and those in the demand scenario.

Initial demand for oxides and intermediate products is then calculated based on the final demand estimates and technology relationships.

### 2.5.2 Initializing Production and Inventories

For all non-oxide products, initial production levels are assumed to match initial demand, and inventories are set to target levels. For oxides, the amount of production coming legally and illegally from each province in China is chosen from within specified ranges using an optimization formulation to match as closely as possible the demand levels multiplied by a set of calibrated parameters, the starting supply target over demand, *sstod*.

The range of possible production levels for each province is taken to be $\left[\frac{BP_{cp,il}}{cpsui}, BP_{cp,il} \cdot cpsui\right]$ for Illegal production and $\left[\frac{BP_{cp,l}}{cpsul}, BP_{cp,l} \cdot cpsul\right]$ is for legal production, where $BP_{cp,il}$ and $BP_{cp,l}$ are province-specific baseline production levels specified in the input data for illegal and legal production. $cpsui$ is the Chinese province supply uncertainty for illegal oxide suppliers, and $cpsul$ is the Chinese province supply uncertainty for mines, with higher uncertainty about illegal production. Inventories are set to be a multiple of target levels, with the multiples being the calibrated parameters starting inventories over targets, *siot*.

Initializing Production and Inventories Parameters

| Parameter | Description |
|---|---|
| *sstod*[1] | Starting supply target over demand |
| $cpsui$ | Chinese province supply uncertainty for illegal oxide suppliers |
| $cpsul$ | Chinese province supply uncertainty for mines |
| *siot*[1] | Starting inventories over targets |

1. Separate values for each oxide.

### 2.5.3 Initializing Prices

For each product, either prices or production costs are specified in the input data. If production costs are specified, starting prices are set based on production costs, and markups, using target markups specified by the target markup parameters $tm$. If prices are specified, production costs (excluding modeled inputs) are estimated to make initial prices consistent with prices from the input data. For metal production we have historical time series price data for both the metals and oxides, which are used to estimate costs and markups through a regression of metal prices against oxide prices. Markups are assumed to be the same for all metals.

Initializing Prices Parameters

| Parameter | Description |
|---|---|
| $tm$[1] | Target markup |

1. Separate values for producer agents.

### 2.5.4 Initializing Substitution

Substitutability parameters and relative costs of alternative substitution options are calibrated to inputs that specify substitution levels at 2010 and 2011 prices, so that there will be a realistic amount of substitution during the historical period.

### 2.5.5 Initial Supplier Shares Optimization

Determining the initial fractions of supply flows from each supplier to each buyer is calculated by an optimization formulation that calculates the shares subject to constraints on supply and demand. The supplier shares must be determined such that each individual buyer flow is satisfied and the total demand flow from each supplier balances the available supply. An algebraic solution is not possible unless all supply and demands are exactly balanced, so an optimization approach is used to minimize the difference between a producer's available supply, and the total demand to that producer from all buyers.

The objective function of the optimization problem is the minimization the total sum of squared error between the supply and demand flows for a product for a set of $I$ suppliers:

$$\varphi = min\left[\sum_{i \in I}(D_i - S_i)^2\right]$$

Each regional supply flow $S_i$ is determined as the fraction of regional supply $r_i$ and total product supply $S$:

$$S_i = r_i S$$

subject to

$$\sum_{i \in I} r_i = 1$$

The total demand on each supplier $D_i$ is calculated as the sum of products of each individual demand $d_j$for $j \in J$ demands, and the supplier share fraction $x_{ij}$ which is the share of supplier $i$ to demand $j$

$$D_i = \sum_{j \in J} x_{ij} d_j$$

Each individual demand $d_j$and each total supply $S_i$ are known fixed quantities, and the supplier shares are free optimization variable subject to the constraints:

$$0 \leq x_{ij} \leq 1$$

$$\sum_{j \in J} x_{ij} = 1$$

The expansion of the objective function leads to the second order form

$$\sum_{i \in I} (D_i - S_i)^2 = \sum_{i \in I} D_i^2 - 2 \sum_{i \in I} D_i S_i + \sum_{i \in I} S_i^2$$

which can be formulated as a quadratic program with linear constraints

$$min \left[ \frac{1}{2} \boldsymbol{x^T P \dot{x}} + \boldsymbol{c^T \dot{x}} + r \right]$$

subject to

$$\boldsymbol{A\dot{x}} \leq \boldsymbol{b}$$

For the objective function given above, we can populate the 1-dimensional solution vector $\dot{\boldsymbol{x}}$ with the unfolded 2D $I \times J$ supplier shares matrix of $x_{ij}$

$$\dot{\boldsymbol{x}} = \left[ x_{11}, x_{12}, x_{13}, \cdots, x_{1J}, x_{21}, x_{22}, x_{23}, \cdots, x_{2J}, \cdots, x_{IJ} \right]$$

The $[1 \times IJ]$ vector $\boldsymbol{c}$ accounts for the coefficients in the objective term $-2 \sum_{i \in I} D_i S_i$ such that

$$c = -2 \left[ S_1 d_1, S_1 d_2, \cdots, S_1 d_J, S_2 d_1, S_2 d_{21}, S_2 d_J, \cdots, S_I d_J \right]$$

and the scalar $r$ is simply the sum of squared supplies:

$$r = \sum_{i \in I} S_i^2$$

The 2D matrix $\boldsymbol{P}$ is a $[IJ \times IJ]$ sparse block diagonal matrix that accounts for the second order coefficients on the solution vector $\dot{\boldsymbol{x}}$

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{q} & \cdot & \cdot & \cdot \\ \cdot & \boldsymbol{q} & \cdot & \cdot \\ \cdot & \cdot & \ddots & \cdot \\ \cdot & \cdot & \cdot & \boldsymbol{q} \end{bmatrix}$$

Each diagonal block $\boldsymbol{q}$ is a $[J \times J]$ dense symmetric matrix defined as

$$\boldsymbol{q} = \begin{matrix} d_1^2 & d_1 d_2 & \cdots & d_1 d_J \\ d_1 d_2 & d_2^2 & \dots & d_2 d_J \\ \vdots & \vdots & \ddots & \vdots \\ d_1 d_J & d_2 d_J & \dots & d_J^2 \end{matrix}$$

<u>Buyer region preferences</u>

The given quadratic program will provide the optimal set of solutions for the supplier shares $x_{ij}$ to minimize supply-demand error for each supplier. The problem formulation does not account for regional buyer preferences to a specific supplier, in that it will determine a mathematically optimal solution without considering that buyers and producers preferentially trade within a region. To account for regional buyer preferences, we specify a regional penalty on the objective function in the form

$$\theta = \sum_{j \in J} \varepsilon_j$$

where $\varepsilon_j$is the objective function penalty for buyer $j$ according to the set of supplier shares $x_{ij}$ for that particular buyer. The penalty function will be large when the buyer prefers in-region and the supplier shares are disproportionally out-of-region. This will cause the optimal solution to maximize supplier shares between buyers and sellers in the same region and minimize supplier shares out-of-region. Each buyer penalty $\varepsilon_j$ is a nonlinear function of the supplier share and regional preference variable $\rho_j$ for buyer $j$:

$$\varepsilon_j = \sum_{i \in I} \begin{bmatrix} x_{ij}(\alpha_i - \rho_j - \rho_j \alpha_i) \ for \ i = j \\ x_{ij}(\alpha_i - \rho_j \alpha_i) \ for \ i \neq j \end{bmatrix}$$

Where $\alpha_i$ is the supply fraction for supplier $i$

$$\alpha_i = 1 - \frac{S_i}{\sum_{i \in I} S_i}$$

The regional preference variable $0 \leq \rho_j \leq 1$ weighs the buyer penalty such that larger values indicates the buyer prefers in-region.

Initial Supplier Shares Optimization Parameters

| Parameter | Description |
|---|---|

| | |
|---|---|
| $\rho_j{}^1$ | Regional Preference |

1. Separate input values for each region.

### 2.5.6 Input Data Adjustment

Much of the model input data has uncertainty ranges specified, and model runs can either be set to pick a central, baseline value for the input data, or to be picked at random from the range. These parameters also allow the size of the uncertainty range to be adjusted. Including uncertainty ranges allows for Monte Carlo sampling of the input space to generate uncertainty ranges for the results through multiple model runs. For each set of inputs with a defined uncertainty range, a parameter allows the user to specify whether to use baseline values for the input data (parameter value 0), the pre-defined uncertainty range (parameter value 1) or a multiple of the pre-defined amount of uncertainty. Uncertainty ranges are defined for the following input data: short and long run prices elasticities of demand, the military share of demand, product prices and production costs, demand growth, regional shares of demand, and regional shares of production, with the corresponding parameters, in the same order: $eur$ (elasticities uncertainty range), $msur$ (military share uncertainty range), $ppur$ (product prices uncertainty range), $dgur$ (demand growth uncertainty range), $dsur$ (demand shares uncertainty range), and $psur$ (production shares uncertainty range).

Input Data Adjustment Parameters

| Parameter | Description |
|---|---|
| eur | Elasticities uncertainty range |
| msur | Military share uncertainty range |
| ppur | Product prices uncertainty range |
| dgur | Demand growth uncertainty range |
| $dsur$ | Demand shares uncertainty range |
| $psur$ | Production shares uncertainty range |

## 2.6 Input data

GCMat input data and sources are summarized in Table 2.2. In this section, we discuss the uncertainties associated with the input data considered in our analysis.

**Table 2.2.** GCMat Input Data and Sources

| **GCMat Input Data** | **Data Sources** |
|---|---|
| US, ROW, China REO mines<br>• Country | ○ USGS (2018): rare earth mining data<br>○ Adamas Intelligence (2016). [supplemented by Adamas Intelligence (2014)] |

| | |
|---|---|
| • TREO content of mine ore (tonnes)<br>• REO concentrations in TREO (%)<br>• Production capacity (tonnes)<br>• Earliest start year<br>• Capital cost (2010$)<br>• Operating cost (2010$)<br>• Other revenues (2010$) | ○ Roskill Information Services Ltd., (2016). [supplemented by Roskill 2011 and 2015 reports]<br>○ TMR (2015) – additional mine data; past year China production quotas<br>○ Varied- publicly available sources, including mining company quarterly and annual reports |
| China production and trade<br>• Chinese export taxes<br>• Chinese production quotas (tonnes)<br>• Illegal TREO production (tonnes) | ○ Adamas Intelligence (2016, 2014).<br>○ Roskill Information Services Ltd. (2011, 2016).<br>○ Curtin-IMCOA (2014a, 2015a-c)<br>○ Ministry of Industry and Information Technology (2016) |
| US, ROW, China producers<br>• 2010 production shares<br>• 2010 production capacities (2010 capacity utilization assumed when capacity data not available) | ○ Curtin-IMCOA (2014b)<br>○ Roskill (2011, 2016): metal refining and magnet production<br>○ Private communications with vendors. |
| Technologies - REO contents | ○ Metals – stoichiometry calculations<br>○ Magnets – sources documented in Riddle et al. (2015)<br>○ Adamas Intelligence (2016, 2014). |
| Technologies – Costs/Prices | ○ Varied - estimated from vendor websites and on-line price data. |
| Rare earth demand estimates and forecasts 2010-2030 | ○ Adamas Intelligence (2016). [supplemented by Adamas Intelligence, 2014]<br>○ Roskill Information Services Ltd., (2016). [supplemented by Roskill 2011 and 2015 reports] |
| Fraction of total demand for defense (defense shares) | ○ SIPRI (2018)<br>○ US DOD (2016): Protected Defense Purchases: Detail by Industry and State |
| US, ROW, China demand shares | ○ US EIA (2017): Table A5 World liquids consumption by region.<br>○ OICA (2016). Sales Statistics<br>○ World Bank (2017)<br>○ Worldsteel Association (2016): True Steel Use |
| Final demand elasticities | ○ Gallaway, M. P., McDaniel, C. A., & Rivera, S. A. (2003). |
| Wind turbine demand (MW/y) | ○ Years 2010-2016: GWEC (2016a, 2016b, 2015, 2014, 2013, 2012, 2011, 2010 ; GWEC 2016 Wind Statistics<br>○ AWEA (2014): US data<br>○ US EIA (2014): US data |

| | |
|---|---|
| | ○ Years 2018-2025: GWEC (2016b): AEO New Policies and GWEC Moderate scenarios |
| Electric and hybrid electric vehicle demand (thousand vehicles/y) | ○ Argonne National Laboratory (2017)<br>○ US EIA (2016): AEO 2016 vehicle sales, subtab_39, downloaded 3/3/17<br>○ IEA (2017)<br>○ IEA (2016)<br>○ Adamas Intelligence (2016)<br>○ IEA (2014) Figure: 1.34 |
| Electric bike demand (thousand bikes/y) | ○ Adamas Intelligence (2016).<br>○ Sources documented in Riddle et al. (2015) |
| Historical rare earth prices for model calibration | ○ Argus Metal Prices<br>○ Federal Reserve Bank of St. Louis (2019): Exchange rates<br>○ US DOC (2019): Producer Price Index Industry Data |

We derived input data uncertainty ranges for final demand, regional production and demand shares, defense shares, product prices, and elasticities as summarized below:

- The uncertainty range for final demand inputs: ranges set to bound historical and forecast estimates from Roskill (2016) and Adamas (2016).
- Regional production shares: ranges estimated from China and US demand for REO by application (catalysts, glass, polishing, metal alloys, magnets, phosphors, ceramics, and other) reported by Curtin-IMCOA (2014b).
- Regional demand shares: ranges informed by minimum and maximum China and US demand shares derived for each final demand product.
- Defense shares: ranges derived from the maximum and minimum defense expenditures as a percentage of GDP reported for China, US, and ROW in SPRI-Milex data.
- Product prices: set multipliers, min = 1; max = 1.25.
- Elasticities: ranges derived from standard errors reported by Gallaway et al. 2003.

# 3 MODEL CALIBRATION AND RESULTS

The goal of calibration for the GCMat model is to determine the input parameter values yielding output values that most closely match historical, real-world price data. The input parameter space of GCMat is too large to exhaustively search for the parameter value combinations resulting in the output values that best fit the historical values. Therefore, the number of input parameters under consideration must first be reduced to a smaller number, and then this parameter space can be searched for parameter combinations and values that yield a good fit. The parameters chosen for the smaller set are those that, when varied, have the largest effect on the model output, in other words, the parameters for which the model is most sensitive.

Model calibration is a process that requires two separate steps - sensitivity and then calibration (best-fit analysis). The large number of individual model runs that are required to accomplish the sensitivity and calibration require a High Performance Computer (HPC) to complete them due to the practical time constraints of running thousands of simulations. The Argonne National Laboratory's Bebop cluster was used for the runs described in this report.

## 3.1 Sensitivity

In the first step of the calibration process, the parameters that have the largest effect on output values are determined. When these parameters are determined, they are used to define a parameter search space of reduced size that can be sampled more effectively for the best fit parameters.

To reduce the number of input parameters under consideration when calibrating GCMat, the Morris method (Morris, 1991) is used in order to rank parameters by sensitivity. The Morris method requires that one or more "elementary effects" be defined, and input parameter sensitivity is determined by how variations in input parameter values affect these elementary effect values. An elementary effect is a value that is a model output value or is derived from a model output value.

For most GCMat Morris method investigations, the elementary effects are the fit error values of the model output price to historical price data. These values are the sums of the errors between the simulated price and the historical price for each time step of the simulation. In GCMat, each model time step represents a week of simulated time.

Determining the most influential inputs is a multi-step process.

For the Morris sensitivity studies, we use the "sensitivity" package available in the R programming language (Iooss et al., 2018). This tool facilitates the generation of parameter values for each run. In GCMat, an Excel spreadsheet is used to define the set of parameters, the base value of each parameter, and the lower and upper limits of the interval within which the parameter values are varied.  Using R, these values, along with the number of parameter intervals to use for each parameter, are input into a Morris object instance from the R sensitivity package. The object generates a list of parameter sets that varies each parameter individually, and covers the minimum/maximum interval span according to the number of intervals. From this, an R script constructs the appropriate parameter files to use in the HPC runs. These parameter files then drive an HPC batch that runs up to a thousand simultaneous individual model instances in highly concurrent sets.

When all of the model runs are complete, the total fit errors for each run are totaled for each price output over the desired model time period. These total fit errors are the elementary effects for the Morris runs. The elementary effect values are input back into the Morris sensitivity R object that originally created the run parameters, and then this object computes each parameter change's contribution to each elementary effect. Summing these contributions over all of the price outputs and sorting the results gives a list of parameters ordered by those that have the most effect on price calibration fit to those that have the least effect.

## 3.2 Calibration

The second step in the calibration process is to perform many individual model runs with a smaller set of varied input parameters. Each run is performed using a different set of values for input parameters and thus will yield different output price data. The runs that are determined to best fit the historical price data are recorded as the best-fit runs, and the parameter values that are used as input to them are saved as the best calibration parameter values.

GCMat is calibrated to historical price data from Metal pages for the following rare earth materials:

- Ce Oxide 99.5-99.9% China (CN) Rmb/mt prices from 200711 to 201703
- Ce Oxide 99.5-99.9% FOB China (CN) $/mt prices from 200201 to 201603
- Cerium metal min 99% ex-works China CNY/mt
- Cerium metal min 99% fob China USD/kg
- Dy metal 99% min China (CN) Rmb/kg prices from 200711 to 201603
- Dy Metal 99% min FOB China (CN) $/kg prices from 200201 to 201603
- Dy Oxide 99.5% min China (CN) Rmb/kg prices from 200711 to 201603
- Dy Oxide 99.5% min FOB China (CN) $/kg prices from 200201 to 201603
- Eu Metal 99% min China (CN) Rmb/kg prices from 200711 to 201703
- Eu Oxide 99.9% min FOB China (CN) $/kg prices from 200303 to 201603
- Europium metal min 99.9% fob China USD/kg
- Europium oxide min 99.99% ex-works China CNY/kg
- Gadolinium metal min 99% ex-works China CNY/mt
- Gadolinium metal min 99% fob China USD/kg
- Gadolinium oxide 99.5-99.9% ex-works China CNY/mt
- Gd Oxide 99% min FOB China (CN) $/mt prices from 200711 to 201411
- La Oxide 99.5-99.9% FOB China (CN) $/mt prices from 200201 to 201603
- Lanthanum metal min 99% ex-works China CNY/mt
- Lanthanum metal min 99% fob China USD/kg
- Lanthanum oxide 99.5-99.9% ex-works China CNY/mt
- Mischmetal 35% La 65% Ce ex-works China CNY/mt
- Mischmetal 35% La 65% Ce fob China USD/kg
- Nd Metal 99% min China (CN) Rmb/mt prices from 200711 to 201603
- Nd Metal 99% min FOB China (CN) $/kg prices from 200201 to 201603
- Nd Oxide 99.5-99.9% China (CN) Rmb/mt prices from 200711 to 201603
- Nd Oxide 99.5-99.9% FOB China (CN) $/mt prices from 200201 to 201603
- Pr Oxide 99.5-99.9% FOB China (CN) $/mt prices from 200201 to 201603
- Praseodymium metal min 99% ex-works China CNY/mt
- Praseodymium metal min 99% fob China USD/kg
- Praseodymium oxide 99.5-99.9% ex-works China CNY/mt
- Praseodymium-Neodymium metal min 99% ex-works China CNY/mt
- Praseodymium-Neodymium metal min 99% fob China USD/kg

- Praseodymium-Neodymium oxide min 99% ex-works China CNY/mt
- Praseodymium-Neodymium oxide min 99% fob China USD/mt
- Samarium metal min 99% ex-works China CNY/mt
- Samarium metal min 99% fob China USD/kg
- Samarium oxide min 99.5% ex-works China CNY/mt
- Sm Oxide 99.5% min FOB China (CN) $/kg prices from 200201 to 201603
- Tb Oxide 99.99% min China (CN) Rmb/kg prices from 200711 to 201703
- Terbium metal min 99.9% ex-works China CNY/kg
- Terbium metal min 99.9% fob China USD/kg
- Terbium oxide min 99.99% fob China USD/kg
- Yttrium metal min 99.9% ex-works China CNY/kg
- Yttrium metal min 99.9% fob China USD/kg
- Yttrium oxide min 99.999% ex-works China CNY/mt
- Yttrium oxide min 99.999% fob China USD/mt

After the parameters that have the largest effect on GCMat output values are determined, the parameters that have the smallest effect on output values are constant in calibration runs since varying their values will have minimal effect on fitting the historical price data. By holding many of the input parameters constant, the model calibration parameter space is significantly reduced. This reduction allows for random sampling of that space for input values and examining how the modeled price output values compare to the historical price values.

To perform a set of calibration model runs, the input parameters are chosen and each of these parameters are given a baseline value, a minimum value and a maximum value. Using these values, Latin Hypercube Sampling (LHS) or Full Factorial designs are used to select samples of parameter values from the input parameter space given. The number of model runs is determined based on HPC run time and output size limits.

As in the Morris case, R scripts are used to generate LHS sample parameter values, and from these values a parameter file is created which contains all parameter value combinations for all runs. The “lhs” function in the tgp (Treed Gaussian Process) R package (Gramacy and Taddy, 2010) is used to generate Latin Hypercube samples of the parameter space.

When all of the HPC runs are complete, price data are combined from all runs into a single file, and calibration statistics are computed from the combined data.

Model output for all runs is analyzed by comparing the modeled weekly price data for each metal or oxide to the historical weekly value in a single run, resulting in the squared error:

$$error_{weekn} = (\log V_{historical} - \log V_{model})^2$$

The logarithms of prices are used so that an equal percentage difference in prices will have the same impact on the fit measure at any historical price level. If the squared difference between the absolute prices (the typical error measure) had instead been used, there would be a greater emphasis on fitting prices when they are highest, i.e., during the peak of the historical price spike.

The error figures for all weeks of the data period in a single run are summed for each oxide/metal, yielding an associated total error figure for each individual run:

$$error_{material} = \sum_{n=1}^{weeks} error_{week\_n}$$

Then these total error figures for each oxide/metal are summed over all material prices in the run to give an overall run error:

$$error_{run} = \sum_{m=1}^{materials} error_{material\_m}$$

The total run error sums from all the HPC model runs are sorted with the lowest total run error chosen as the best fit run overall.

A series of calibration model runs was conducted using high-performance computing workflows implemented with the EMEWS framework (Ozik, Collier, Wozniak, and Spagnuolo, 2018). EMEWS is built on the general-purpose parallel scripting language Swift/T (Wozniak et al. 2013), which provides the capability of running multi-language software tasks on anywhere from desktops to peta-scale plus computing resources. EMEWS enables running very large, a priori defined parameter sweeps such that each parameter combination is provided to an individual GCMat Repast model instance running in a separate Java runtime.

A total of 10,000 individual model runs was executed on the Bebop cluster in batches of 1,024 simultaneous runs concurrently across 32 compute nodes, with 32 processes per node. The total real wall time to complete all 10,000 runs is approximately 9.5 hours.

Figures 3.1-3.8 show example analysis output from those runs. In the figures, the black trace is the historical price data. The green traces are the top runs that best fit the historical data for all prices. The red traces are the bottom runs that worst fit the historical data for all prices. Plots similar to these are generated for all historical material prices during the calibration process.

Figure 3.1 shows the ten best-fitting and worst-fitting calibration runs for a single price. Note that the "best" and "worst" rankings are based on the cumulative error over all prices, not simply the price for the graph shown. Figure 3.2 shows a similar price plot as Figure 3.1, but only showing the single overall best-fit calibration run data against the historical data. Figure 3.3 and Figure 3.4 show similar information, but for a different element price.

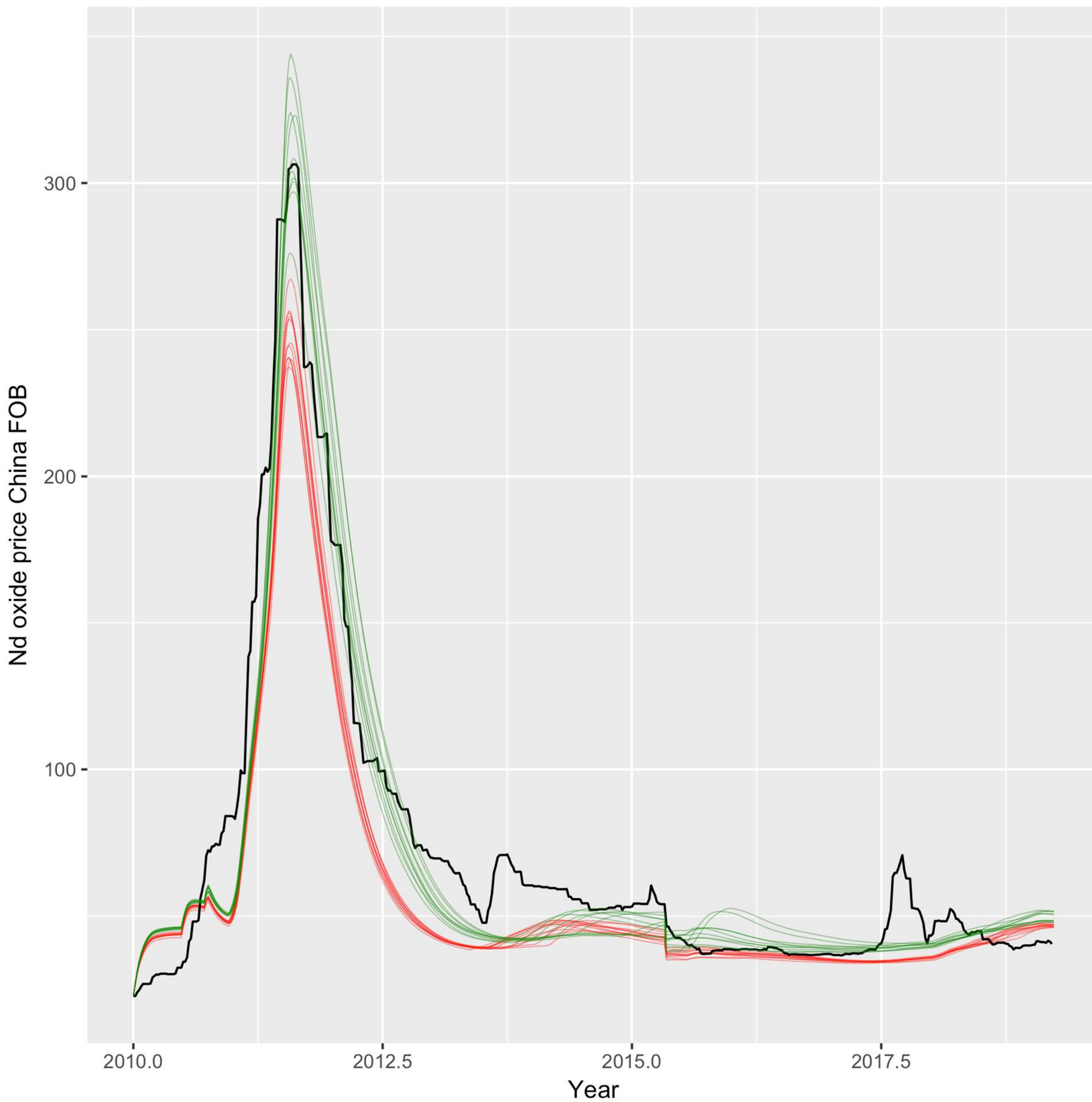



**Figure 3.1. Top Ten Best and Worst Fitting Calibration Runs (Nd Oxide China FOB)**

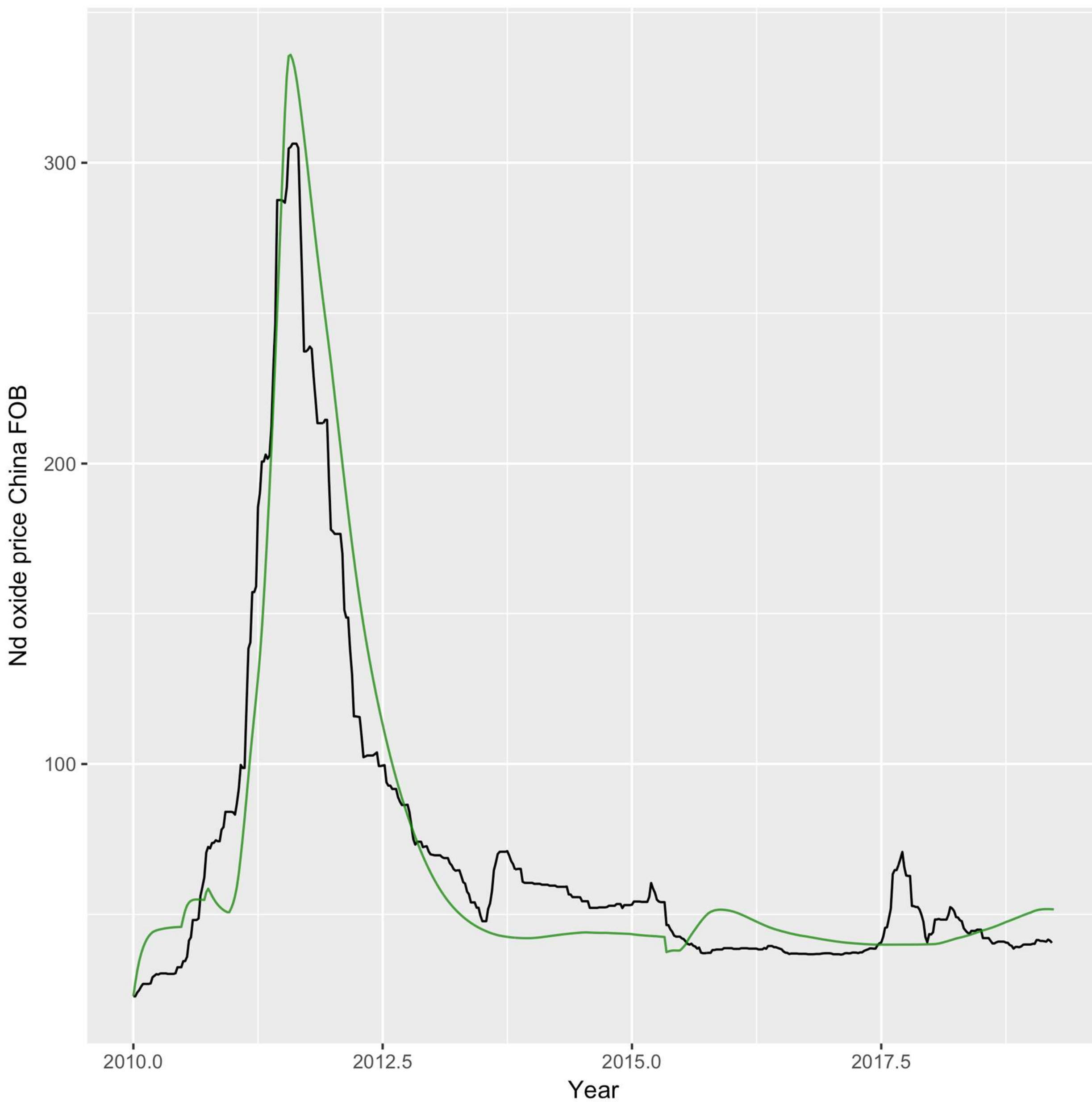



**Figure 3.2. Overall Best Fit Run vs. Historical Data (Nd Oxide China FOB)**

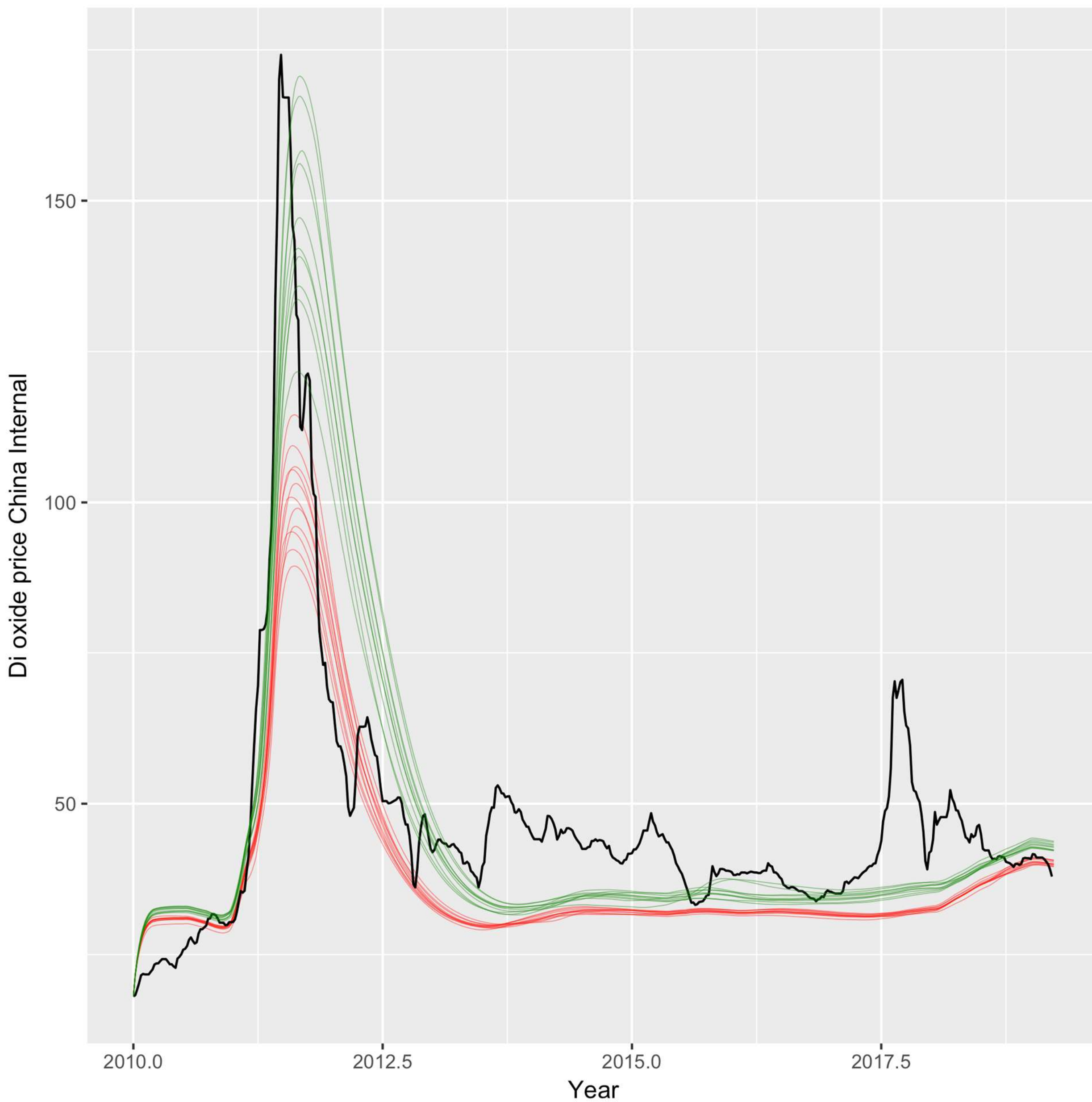



**Figure 3.3. Top Ten Best and Worst Fitting Calibration Runs (Didymium Oxide China Internal)**

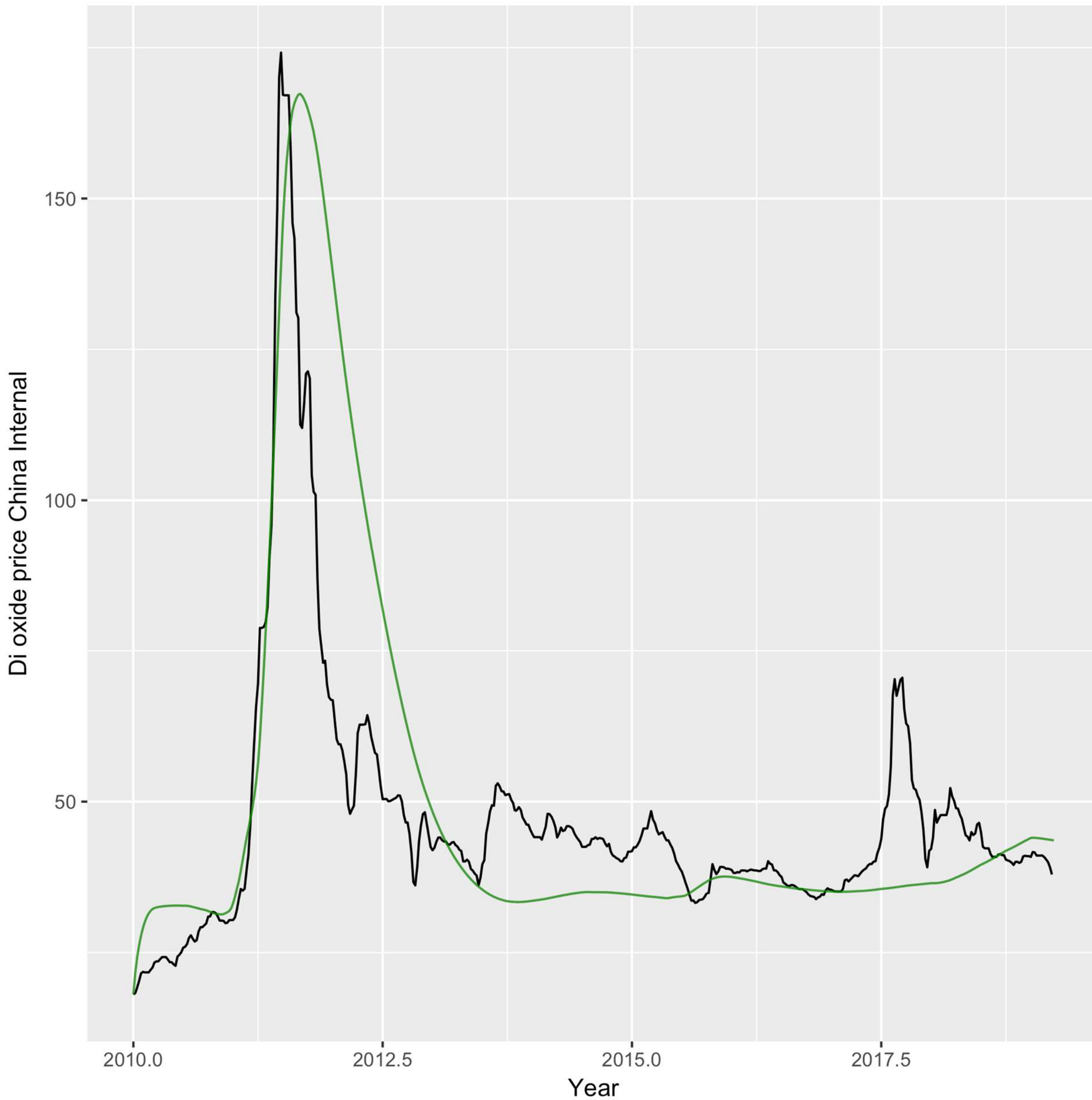



**Figure 3.4. Overall Best Fit Run vs. Historical Data (Didymium Oxide China Internal)**

To ascertain overall fit across all runs, Figure 3.5 shows the price fit error for each run, ordered by best fitting to worst. The run depicted at the very left, is the overall best fitting run and the run at the very right is the worst fitting run.

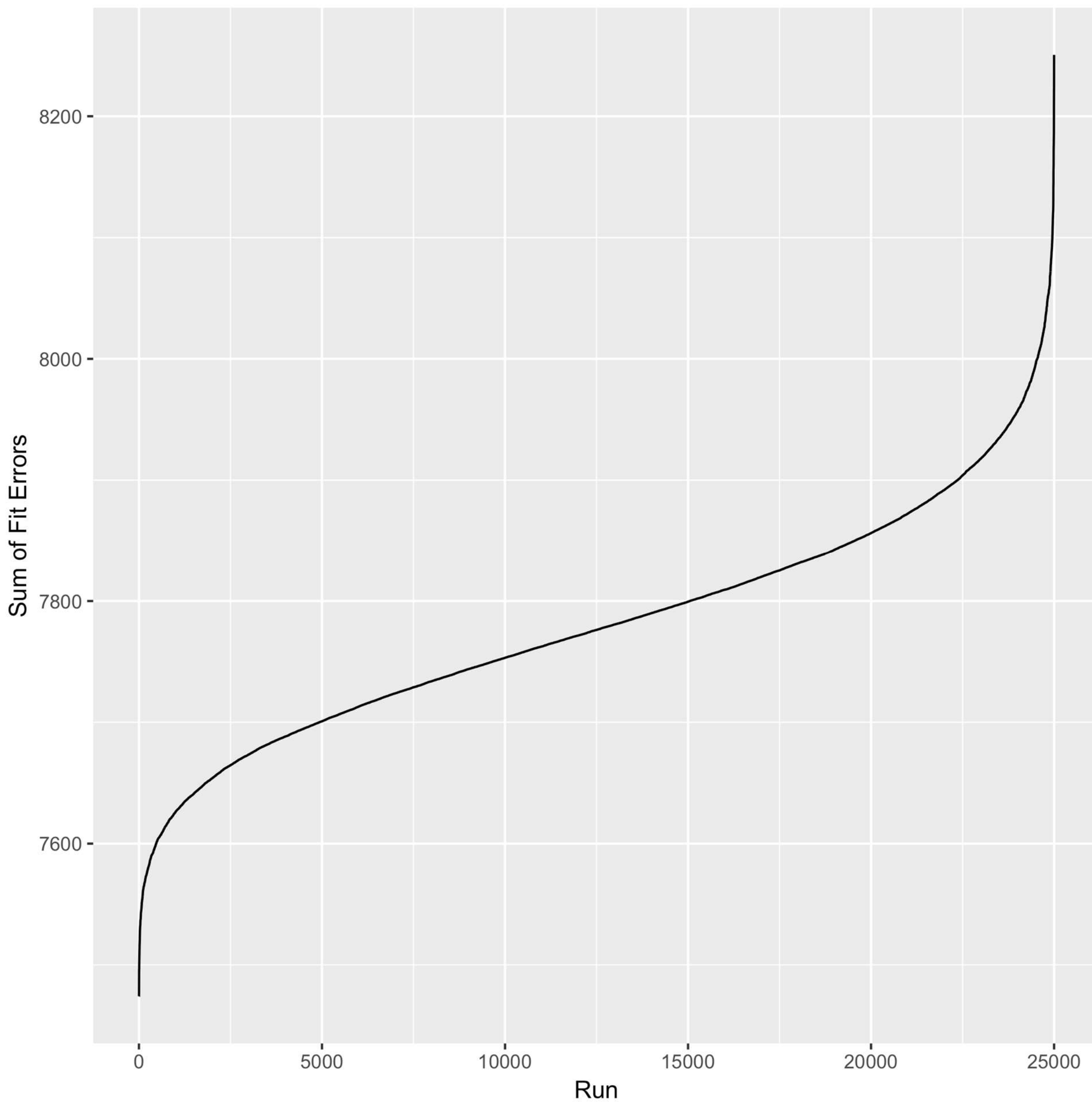



**Figure 3.5. Sum of Fit Errors For All Calibration Runs**

Figure 3.6 shows the distribution of total fit errors across all runs.

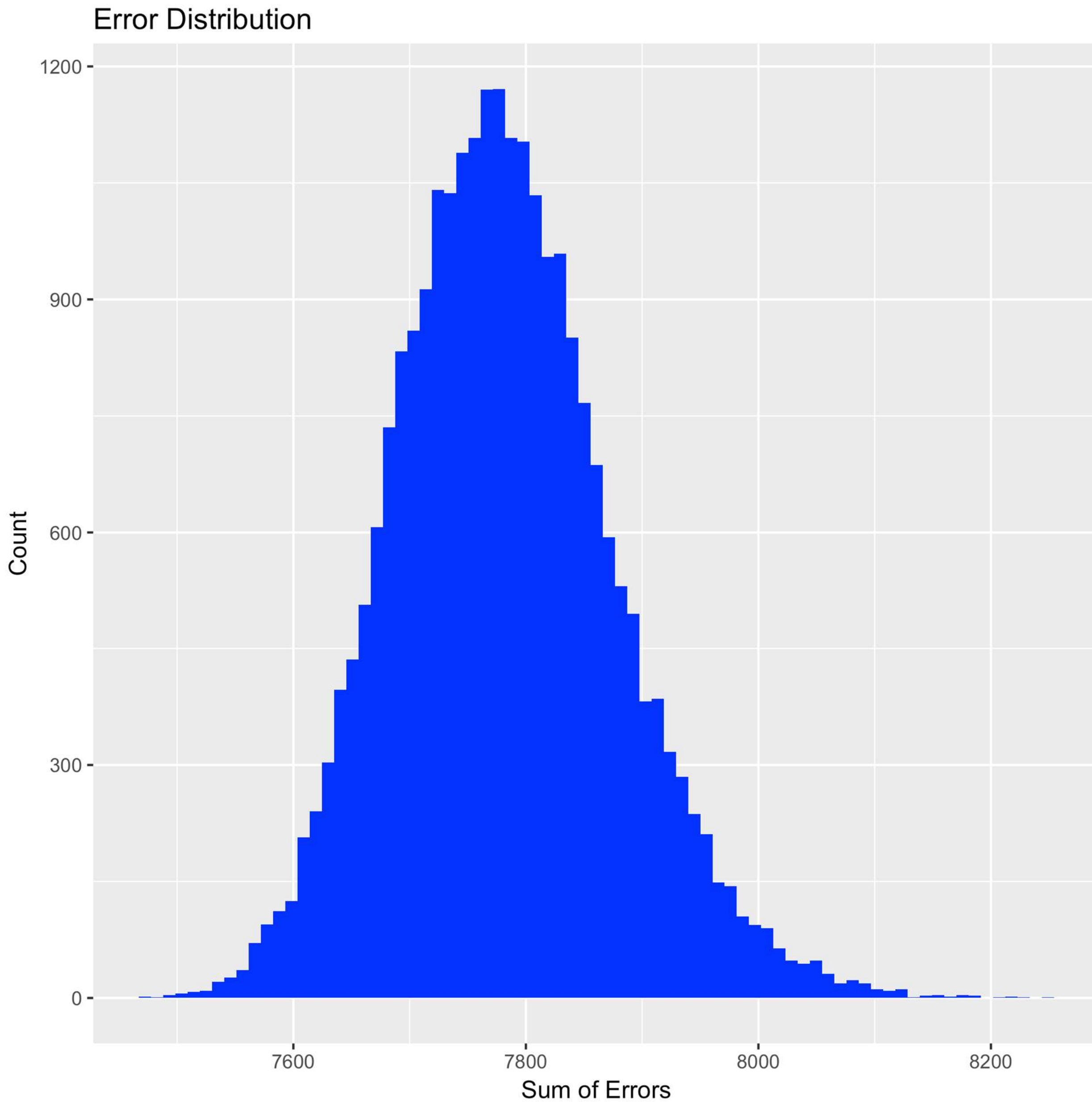



**Figure 3.6. Distribution of Fit Error Sums**

The distribution of parameter values for the best-fitting runs are also examined. In these, the top 25 best-fit runs have the parameter values for a particular model parameter plotted as a histogram of all the values that the parameter took in those runs. Figure 3.7 shows a parameter where 23 of the best fit runs occurred with the parameter value with the values .857-.861 inclusive, while only 2 of the best fit runs occurred with the value less than that range.

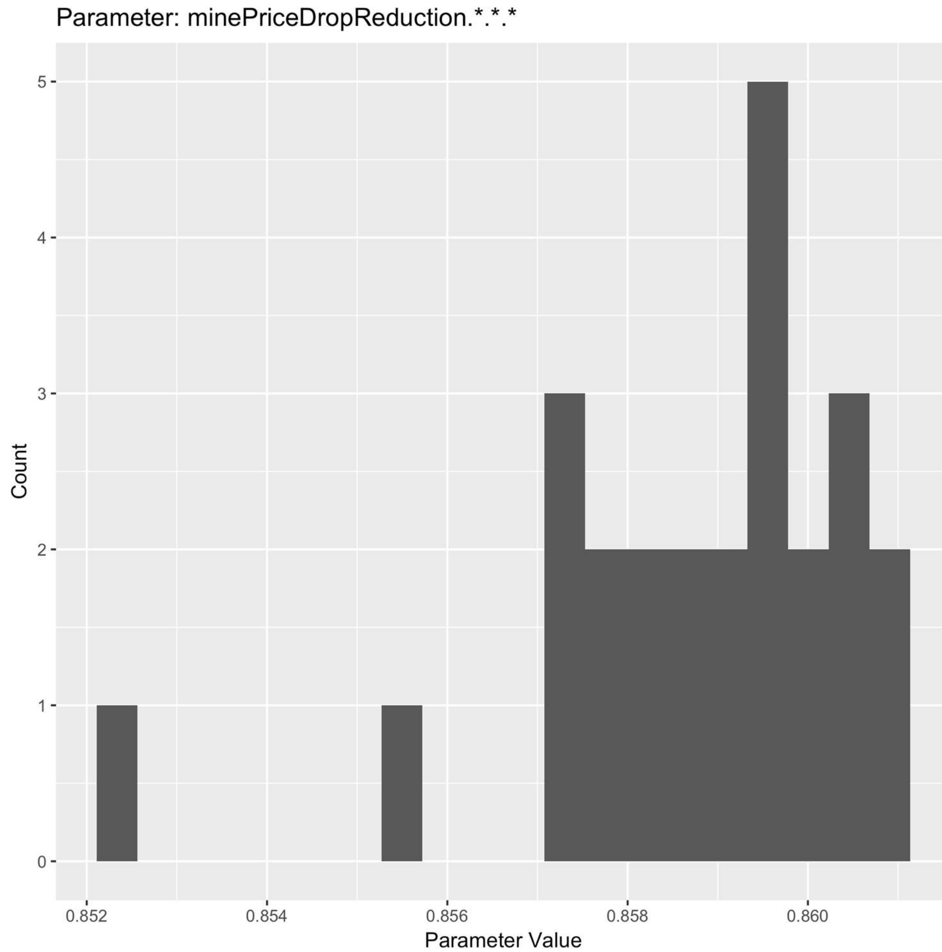



**Figure 3.7. Single Parameter Value Distribution for Top 25 Runs**

Figure 3.8 shows a parameter where the best fit runs occurred with no particular pattern to the parameter value.

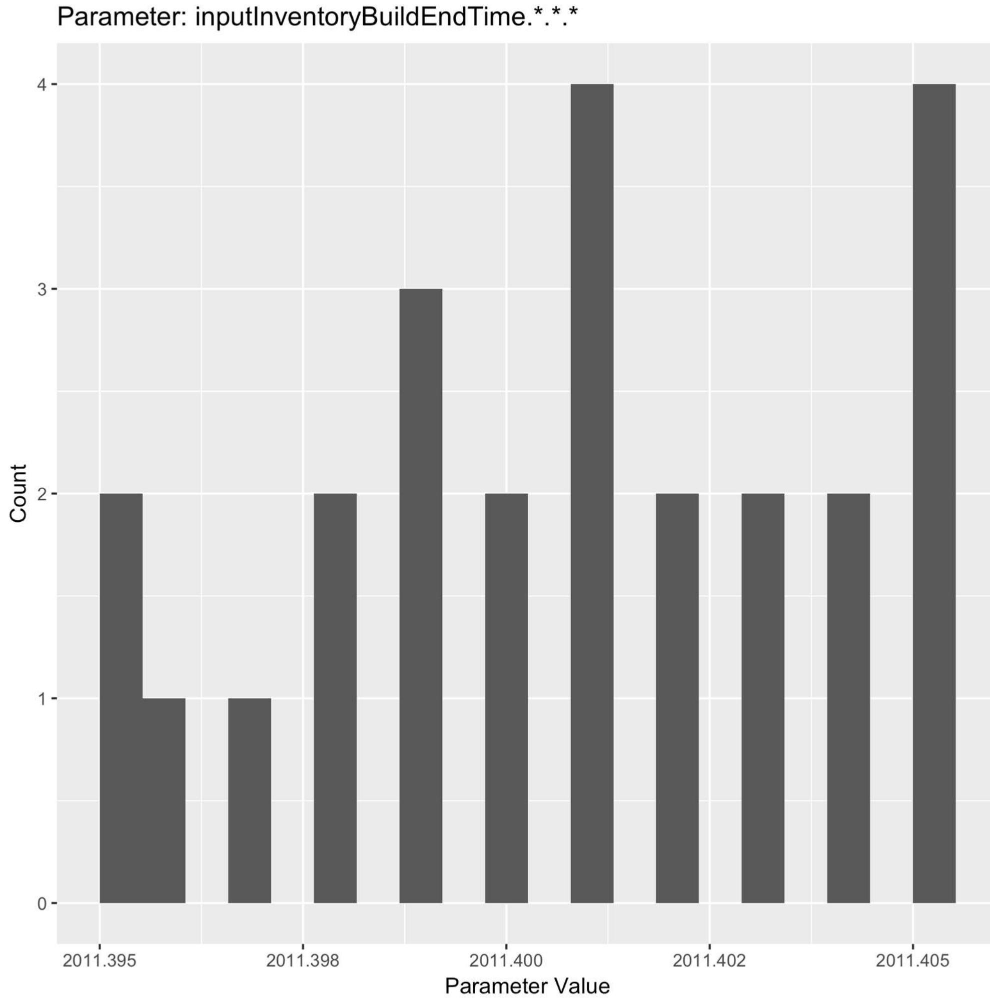



**Figure 3.8. Single Parameter Value Distribution for Top 25 Runs**

# 4 SAMPLE RESULTS

In this section, we provide results for a baseline scenario. Baseline results shown in Figure 4.1 illustrate that our calibrated model replicates historical patterns from 2010–2018 and generates plausible projections of future developments from 2019–2030.

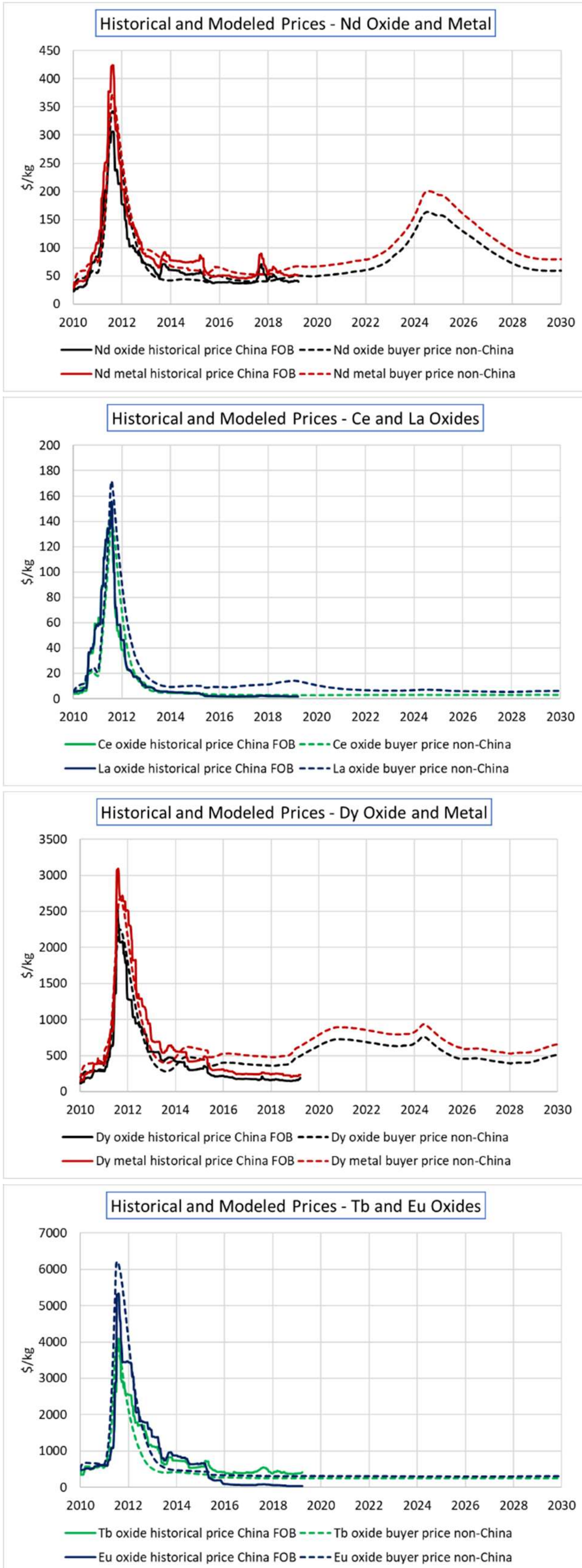



**Figure 4.1. Rare earth oxide price results from baseline scenario run with comparison to historical**

In addition to rare earth oxide and metal prices, the GCMat simulation generates many other results, including week by week results for each mine, producer, and final demander agent covering prices, production, capacities, inventories, sales, purchases, contracts, imports, exports, and RE oxide contents of all products. Output files are listed and briefly described in Table 4.1.

**Table 4.1.** GCMat output files

| File name | Description |
|---|---|
| GUIOutputs.csv | Weekly time series data used in graphs shown in GUI. See DataSeries tab for details. |
| MediumOutputsV2.csv | Additional weekly time series data. See DataSeries tab for details. |
| demand.csv | At each tick, final annualized demand of each product (in product units) by buyer region |
| development_status_change_events.csv | Records of when deposit development is initiated, abandoned, completed (and production begins), or a producing deposit is shut down. Time tick of change, facility, and change type are recorded. |
| development_status_events.csv | At each time tick, records the operating status and TREO, Nd oxide, and Dy oxide capacity of each mine. |
| final_sales_events.csv | At each time tick and for each product sale, records the supplier and buyer names and regions; product name, demand, and sales; and Nd and Dy oxide contents for base demand, simulated demand, and sales. |
| inventory_change_events.csv | Detailed records of every time inventory levels are changed, including new inventory level and inventory targets. We can provide post-processing at a future time to translate these record into a more readable format. |
| magnet_technologies.csv | At each time tick and for each magnet producer agent, records the technology and Nd and Dy content for the magnet produced. |
| offers | At each time tick, record of offers, specifying buyers and suppliers by region and legality, quantity, offer price, and buyer price. |
| oxide_extraction_events | At each time tick and for each mine deposit, amount produced and production capacity of total TREO and individual REOs in tonnes |
| production_events.csv | Detailed records of weekly production by all agents, including production capacity at the time of production. |
| transaction_events.csv | Detailed records of simulated sales transactions, including supplier and buyer names and regions; contract type and legality; demand, sales, supplier price (before tax), buyer price (after tax), price offer, export tax rate, and Nd and Dy content of sales. |
| unmet_final_demand_events.csv | Detailed records of each time a demand request is not fully met by a supplier, including supplier, buyer, product, amount of unmet demand, and Nd and Dy oxide content of that unmet demand. |
| nd_dy_price_production.csv | Yearly averages and yearly standard deviations for oxide and metal prices and unmet demands. |
| nd_dy_stats.csv | Calculated fits to historical data for 8 price series, as well as mins, maxes and std. devs. |

**ProcessDataRunner outputs**

| File name | Description |
|---|---|
| inventoriesByProductAndType.csv | At each tick, reports the total (global) input and product inventories, in product units, for each producer type. |
| inventoryChangesAndTargetsByProductTypeAndRegion.csv | At each tick, reports the China, US, and ROW addition, subtraction, new, and target input and product inventories for each product |
| productBuyerPrices | At each tick, reports for each product the weighted average yearly sales (in product units) by supplier region, buyer region, and legal/illegal supply. |
| productSupplierBuyerBuyerPrices.csv | At each tick, reports for each product the weighted average prices (in product units) by supplier region, buyer region, and legal/illegal supply. |
| productSupplierBuyerSales.csv | At each tick, reports for each product the weighted average yearly sales (in product units) by supplier region, buyer region, and legal/illegal supply. |
| productionAndCapacitiesByProductAndRegion.csv | At each tick, reports China, US, and ROW annual production capacity and annualized amount produced for each product. |
| productionAndCapacitiesTREOByDeposit.csv | At each tick, reports annual production capacity and production (tonnes) for each mine deposit |
| OxideContentsBaseDemandAndSalesByProduct.csv | At each tick and for each individual REO, reports REO content totals for sales, sales with initial oxide content, and base demand with initial oxide content for all products. |
| OxideContentsBaseDemandAndSalesTotal.csv | At each time tick and for each individual REO, reports total REO content in sales, sales with initial oxide contents, and base demand with initial oxide contents |
| magnetMetalContentsByTechnology.csv | At each tick and for each magnet type, reports weighted average simulated and base fractions of Nd, Dy, Pr, and didymium metals in magnets |

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